

# **Nemeth Uniform Braille System**

## **Instruction Manual for Braille Transcribing**

### **Part Two: Scientific Notation**

**The Alliance for Braille Literacy**

**2024 Edition 9/1**

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*In honor of*  
*Abraham Nemeth and Joyce Hull,*  
*this manual is dedicated to all blind students and adults.*



## **NUBS Course for Scientific Notation**

This manual is Version 3 provided in 2024. If there are changes and or corrections needed, please contact ABL at the web site from which you received this.

This Course for Scientific Notation covers topics from the less complex to the most complex. The rules presented in the manual for Literary NUBS are retained in this manual; NUBS is a single, complete braille system.

The material presented in these sections is related to new symbols, to formatting, and descriptions of how to apply the basic rules to science concepts. There are a number of examples for each topic. Appendices provide lists of all defined symbols and there are also rules available for devising undefined symbols as needed.

It is necessary for you to have completed the Literary Course for NUBS before starting this course about scientific notation using NUBS. We recommend that you review Chapter 14 before you begin this course as the information there is basic to all that this manual presents.

The first section gives a description of how notational phrases are used. Material is presented so that everything needed for a particular subject is in one section. For example, everything needed to handle addition and subtraction, including any special rules for formatting, is in Chapter 21. Chapter 22 covers Multiplication and Division in the same manner.

Topics are covered in ten chapters numbered 20 to 29. Chapters 21 through 27 have one or more drills for practice. Chapters 20 through 26 have Exercises at their ends that should be sent to the instructor for review. After successful completion of an exercise, the student is ready to continue with the following chapter. Chapters 28 and 29 deal with advanced topics and give examples of how to apply the rules to the numerous symbols used in higher mathematics. There are no drills or exercises for these two chapters. Students should at this point be able to apply what has been learned.

The chapters are arranged so that the less-complex topics are described before those that cover higher level subjects. A transcriber should study topics up to and through those that pertain to their needs.

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# Lesson 20

## Rules Refresher

### 20.1 In General

Everything you learned in the Literary Course of NUBS applies to the math and science aspects. This is all one braille system. The information here is based on the NUBS Literary Manual, especially Lesson 14; we recommend reviewing Lesson 14 before moving ahead in this course.

### 20.2 Notational and Narrative Indicators

Numeric Indicator (number sign) ⠆ (3456)

Notational Indicator ⠆ (56)

Notational Phrases

Open if phrase begins with a digit ⠆⠆ (56,3456)

Open if phrase begins with a non-digit ⠆⠆ (56,56)

Close phrase in both cases ⠆⠆ (56,3)

### 20.3 Notational Punctuation

Period ⠆ (12456)

Colon ⠆ (156)

Mode switch (dot-5 switch) ⠆ (5)

Punctuation Indicator ⠆ (456)

Left Parenthesis: Narrative ⠆⠆⠆ (456,12356)

Notational ⠆⠆⠆ (56,12356)

Right Parenthesis: Narrative ⠆⠆⠆ (456,23456)

Notational ⠆⠆ (23456)

Opening and closing parentheses are clearly identified as such in NUBS so there are no restrictions on what words may be in contact with parentheses. Rules for using parentheses are found in 12.2.2 and 15.6 of the NUBS Literary Manual.

### 20.4 Notational Phrases

Any math statement of three or more notational words will be placed between the notational phrase indicators. These are listed above. A notational phrase ensures that all the words are readily understood as being notational and that spaces between them do not terminate the notational mode.







The dot-5 switch is required in Roman or Arabic numbers that are followed by ordinal endings, st, nd, rd, th. Contractions can be used in English ordinal endings, but not with endings in other languages.

Examples:

10th ⠠⠠⠠⠠⠠⠠      xth ⠠⠠⠠⠠      21<sup>st</sup> ⠠⠠⠠⠠⠠⠠⠠  
 XXI<sup>st</sup> ⠠⠠⠠⠠⠠⠠⠠⠠⠠      2<sup>nd</sup> ⠠⠠⠠⠠⠠⠠      3<sup>rd</sup> ⠠⠠⠠⠠⠠⠠  
 I<sup>er</sup> (French) ⠠⠠⠠⠠⠠⠠      VI<sup>e</sup> (French) ⠠⠠⠠⠠⠠⠠⠠  
 5ten (German) ⠠⠠⠠⠠⠠⠠

## 20.9 Decimals

Since a decimal point in print is identical to a period, the same is true in NUBS. Note that if the number starts with a decimal point, a notational indicator must precede it rather than a numeric indicator. This is because a decimal point, though notational, is not a digit.

Examples:

.7 ⠠⠠⠠      .03 ⠠⠠⠠⠠⠠      .5 - .7 ⠠⠠⠠⠠⠠⠠⠠

When a number consists of a whole number and a decimal part, only one numeric indicator is required, and it is placed before the whole number.

Examples:

90.9 MHz ⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠      0.5-1.2 ⠠⠠⠠⠠⠠⠠⠠⠠⠠  
 6-7.25 ⠠⠠⠠⠠⠠⠠⠠⠠      7.25-8 ⠠⠠⠠⠠⠠⠠⠠⠠

## 20.10 Proportions

As with other signs of comparison, any math statement of three or more notational words will be placed between the notational phrase indicators. Those are listed with other indicators at the beginning of this lesson. These ensure that all the words of the phrase are readily understood as being notational.

In mathematics, a structure is needed to indicate that two quantities have a relationship which is the same as that for two other quantities. For example,





### 20.13 Units of Measure

For units of measure, do not use the rules for abbreviations and acronyms. Instead use the following rules:

1) If the unit of measure is not in contact with any digits, and if it stands separate from other abbreviations, it is narrative. If two or more abbreviations with periods between them are unspaced from each other, they are notational. Another special case is noted under number 3 below.

Examples:

ft. ⠠⠠⠠⠠    mL ⠠⠠⠠⠠    hr. ⠠⠠⠠⠠  
sq. ft. ⠠⠠⠠⠠ ⠠⠠⠠⠠    sq. ft. ⠠⠠⠠⠠⠠⠠⠠⠠⠠  
cu.yd. ⠠⠠⠠⠠⠠⠠⠠⠠⠠

2) If the unit of measure is unspaced from one or more preceding digits and,

a) if the unit of measure is notational, the word is notational,

Examples:

2sq.ft. ⠠⠠⠠⠠⠠⠠⠠⠠⠠

We dug 50cu.yd. of sand. ⠠⠠⠠⠠ ⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠ ⠠⠠ ⠠⠠⠠⠠

b) if the unit of measure is narrative, the word is a hybrid. Remember, a hybrid word is one in which part of the word is notational and part is narrative. In a hybrid word that begins in the notational mode, a dot-5 “switch” is required before the narrative part, and the rest of the word is narrative.

Examples - hybrid:

5mL ⠠⠠⠠⠠⠠⠠⠠⠠⠠ 5KW ⠠⠠⠠⠠⠠⠠⠠⠠⠠

2yds, 4ft. ⠠⠠⠠⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠⠠⠠⠠









8. Here are a few examples of ordered pairs.  $(x,y)$   $(a, b)$   $(3,4)$
9. Can you name this number? 123,456,789,012,345
10. 987,654,321,123,456,789,987,654,321,123,456,789 is a huge number.
11. How many feet in 2 yards?
12. 9 sq.ft. is 1 sq. yd.
13. His appointment is set for 2:15 p.m.
14. School will be closed from 11/22–11/25 for Thanksgiving.
15. Our school team won the game 42-34.
16. She won the election by 256 votes, 22,346—22,090
17. The ABL website is at [www.allforbraille.org](http://www.allforbraille.org).
18. There are 6-8 cherries in each bag.
19. For you, there are 2—3 to give away.
20. Monday September 11, (9/11/2001) was a very sad day in the USA.





### 21.3.1 Linear Format, Addition and Subtraction

Many simple addition and subtraction problems are shown in a linear format in print.

Examples:

$$2+3 = 5$$

$$39-26 = 13$$

$$36+25 = 61$$

$$117-99 = 18$$

$$63¢ + 21¢ = 84¢$$

Notice that in each of these examples the numeric phrase indicators are needed before the first number and after the last number, but no indicators are used either before or a after the equal sign. When a letter or other symbol is the first character of an equation, two notational indicators are used instead of one notational indicator with a numeric indicator.

Examples:

$$\$6.92 + \$0.44 = ?$$

$$\$8.35 - \$0.56 = \$7.79$$

$$x + y + z = 455$$

$$y + 2y = 3y$$

$$-? + 5 = 12$$

### 21.3.2 Summary of Rules for Linear Format

- A. A beginning notational phrase indicator (56,3456) must precede any digit that is at the start of a math problem consisting of three or more notational words.

- B. A beginning notational phrase indicator ⠠⠠ (56,56) must precede any non-digit that starts a math problem consisting of three or more notational words.
- C. A closing notational indicator ⠠⠠ (56,3) must follow the last word of the notational phrase.
- D. Spaces are required before and after any sign of comparison, such as an equal sign or greater than or less than symbol. These spaces do NOT end the notational mode because the problem is within a notational phrase.
- E. No spaces are left before or after any sign of operation, in this case plus or minus signs.
- F. If there are italics in a print problem, they should be ignored.
- G. When a notational phrase is used, any identifier for an example may optionally be included within the phrase. The rules described above apply to the example, with the notational phrase indicator being placed either before the identifier or before the example itself.

Examples:

23 plus 326 equals 349

$$23 + 326 = 349$$

10 minus 8 is less than 6

$$10 - 8 < 6$$

1. a plus b plus 6 is greater than a plus b

$$1. a+b+6 > a+b$$

## 21.4 Notational Comma Use

A comma within a number must be in the notational mode. For example, in the number 1,000, if a narrative comma were used, the number would read as 11000 because the narrative comma would be mistaken for the number 1. The notational comma ⠠ (16) is used instead to avoid confusion.

Example:

$$2,567 \quad 10,245$$

## Drill #1

1. In 1,234,567, the 1, 3, 5, and 7 are odd numbers.
2. Add: 30, 40, 60 and 70.
3. What does  $15+6+150$  equal?
4. If  $4+6 = 10$  then does  $10-6 = 4$ ?
5.  $\$3.50+\$3.50 = \$7.00$
6.  $50¢-35¢ = 15¢$
7.  $.7 > .3$
8.  $-.32+.98 = +.66$
9.  $568+297 = ?$
10. Round to the nearest tenth: 9.45, 3.86, 423.55

### 21.5 Spatial Display for Addition and Subtraction

Braille is designed by its very nature to be a linear presentation of text. This serves well in normal literary applications, but it leaves students unaware of the actual form used in print for simple arithmetic problems. Braille systems solve this with a format called spatial display.

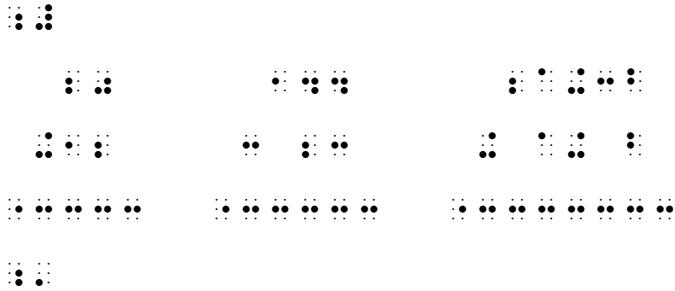
Spatial display is used for material that is set off in print from the rest of the text by blank lines above and below the text. Examples in a non-linear format and materials that involve indentation, margin changes, and/or columns or tables are also handled with spatial display.

Consider the examples that are shown below. In print they are set off as displayed text, and spatial display allows braille readers to have the same information as sighted students. Notational phrase symbols are placed at the beginning of the lines above and below the displayed text.

Note the ruled lines, as shown in the braille below; a dot 5 followed by a series of dots 25. The lines extend one space to the left and one space to the right of each problem.

Examples:

$$\begin{array}{r} 20 \\ +12 \\ \hline \end{array} \qquad \begin{array}{r} 144 \\ - 23 \\ \hline \end{array} \qquad \begin{array}{r} 2a+3b \\ + a + b \\ \hline \end{array}$$



### 21.5.1 Format issues for spatial display

- A. Above the text, there is a blank line with only an open notational phrase indicator placed at its start. Below the text, there is a blank line with only a close notational phrase indicator placed at its start. If the last line of the text is on the last line of a page, the close notational phrase indicator may be placed at the end of that last line instead of on the next line. A blank line should then be left at the top of the next page.
- B. Entries must be aligned according to their decimal value or by their letters unless they have been intentionally misaligned as an exercise for the student.
- C. For addition and subtraction, plus or minus signs must appear at least one column of cells to the left of the widest column of numeric symbols which appears in the part of the arrangement above the ruled line.
- D. The ruled line must extend one cell beyond the left and right extremes of each problem.
- E. Operators and variables within a problem, as in the second example below, must be aligned.
- F. When spatial arrangements are placed side by side across a page without identifiers, at least two blank spaces must be left between the end of one ruled line and the beginning of the next.
- G. When examples have identifier numbers or letters, they are placed on the same level as the ruled lines. It is not required for identifier numbers to be preceded by either a notational indicator or a numeric indicator because they are part of the notational phrase; optionally, the transcriber may use notational indicators on the identifiers. A note



$\begin{array}{r} 476 \\ +13 \\ \hline 489 \end{array}$	$\begin{array}{r} 273 \\ +24 \\ \hline 297 \end{array}$	$\begin{array}{r} 127 \\ 231 \\ +30 \\ \hline 388 \end{array}$	$\begin{array}{r} 952 \\ 123 \\ +11 \\ \hline 1,086 \end{array}$
4.	5.	6.	7.

## 21.6 Carried Numbers in Addition

In spatial addition problems, the number carried over to the adjacent column on the left is often shown **in reduced size** in print. Digit alignment is essential.

Example:

1 2 1	• • •	
662	• • • • •	<b>carried number line</b>
1075	• • • • •	
+ 974	• • • • •	
2711	• • • • •	<b>ruled line</b>
	• • • • •	

Note that a “carried number line” (dots 56, and a series of dots (2356) is placed between the top element of the addition and the carried numbers. This line is two cells shorter than the ruled line, and positioned as shown above.

Each carried number is placed above the column to which it is to be added.

The **carried number line** will also be used as the **borrowed number line** in subtraction as shown below.

## 21.7 Borrowing (Renaming) Numbers in Subtraction

It is clear at a glance that the example below does not require any “borrowing” or “renaming” because each of the digits being subtracted is less than the digit above it. In the problem below, the subtraction can be carried out easily so there is no need for carrying any numbers.

$$\begin{array}{r}
 1456 \\
 - 123 \\
 \hline
 \end{array}$$

The next example does require some renaming. It can be seen at a glance that 7 cannot be subtracted from 4, so we need to "borrow" a 1 from the 9, changing it to an 8 and replacing the 4 with 14. In print, this is no problem, in that the number 14 can be written above the 4 in smaller font size, so that the alignment of the columns is maintained. This is shown on the right, in print. As seen below, braille doesn't allow the 14 to align very well over the 4.

$$\begin{array}{r}
 1394 \\
 - 267 \\
 \hline
 \end{array}$$

### 21.7.1 Replacement Number Technique

NUBS solves this subtraction process by providing a clever technique.

Fortunately, a two-digit replacement number in a subtraction scheme is always in the range of 10 to 18 inclusive. Therefore, we are able to devise a technique for entering this two-digit number into one cell. We consider the unit's digit of this number as a dropped number, as usual. We now add dot 1 into the same cell to indicate that this is a two-digit number whose ten's place is 1. Thus, the numbers from 10 to 18 would be represented in one cell as follows:

⠠	⠠	⠠	⠠	⠠	⠠	⠠	⠠	⠠
10	11	12	13	14	15	16	17	18

Place a borrowed number line, as described above, to separate the borrowed numbers from the problem. This line should be one cell shorter on each end than the ruled line. Using this technique, a problem requiring borrowing would be brailled as:

$$\begin{array}{r}
 1394 \\
 - 267 \\
 \hline
 1127
 \end{array}$$

This technique does not require cancellation indicators. For subtraction this is all we need to know for cancellation. Here are a few more examples to help you get comfortable with the technique. Note: It can be useful to use a second level of cancelled numbers as in the second example, or it may be handled with just one level depending on the understanding of the student. Also note that these two-digit numbers look the same as some letters and other symbols; when placed above the borrowed number line they instead represent one of the numbers from 10 through 18.

Examples:

$  \begin{array}{r}  572 \\  - 365 \\  \hline  \end{array}  $	$  \begin{array}{r}  231 \\  - 89 \\  \hline  \end{array}  $	$  \begin{array}{r}  1,246 \\  - 159 \\  \hline  \end{array}  $

Remember, if a cell above the carried/borrowed line, contains a dot 1, that cell represents one of the teen numbers. This system really does simplify the demonstration of subtraction problems that require borrowing and renaming.

## Drill #3

Use carried and borrowed numbers where needed.

$$1. \begin{array}{r} 246 \\ -79 \\ \hline \end{array}$$

$$2. \begin{array}{r} 387 \\ +146 \\ \hline \end{array}$$

$$3. \begin{array}{r} 1,256 \\ +2,576 \\ \hline \end{array}$$

$$4. \begin{array}{r} 525 \\ -28 \\ \hline 497 \end{array}$$

$$5. \begin{array}{r} 273 \\ -84 \\ \hline 189 \end{array}$$

$$6. \begin{array}{r} 127 \\ 231 \\ +56 \\ \hline 414 \end{array}$$

$$7. \begin{array}{r} 958 \\ 123 \\ +47 \\ \hline 1,128 \end{array}$$

## Exercises

Practice brailleing these examples of displayed addition and subtraction. Use carrying and cancellation notations where needed.

1.  $\$4.77 + \$3.22 = \$7.99$

2.  $.09 + .22 = .31$

3.  $368 - 255 + 134 = ?$

4. Is the following true?  $1.29 + 2.55 - .65 > 3.25$

$$5. \begin{array}{r} 134 \\ +15 \\ \hline 149 \end{array}$$

$$6. \begin{array}{r} 252 \\ +46 \\ \hline 298 \end{array}$$

$$7. \begin{array}{r} 127 \\ +62 \\ \hline 189 \end{array}$$

$$8. \begin{array}{r} 958 \\ +63 \\ \hline 1,021 \end{array}$$

$$9. \begin{array}{r} 236 \\ 192 \\ +45 \\ \hline 473 \end{array}$$

$$10. \begin{array}{r} 519 \\ 228 \\ +61 \\ \hline 808 \end{array}$$

$$11. \begin{array}{r} 473 \\ 446 \\ +29 \\ \hline 948 \end{array}$$

$$12. \begin{array}{r} 2,356 \\ +678 \\ \hline 3,034 \end{array}$$

$$13. \begin{array}{r} 357 \\ -15 \\ \hline 342 \end{array}$$

$$14. \begin{array}{r} 876 \\ -71 \\ \hline 805 \end{array}$$

$$15. \begin{array}{r} 2,379 \\ -268 \\ \hline 2,111 \end{array}$$

$$16. \begin{array}{r} 5,674 \\ -361 \\ \hline 5,313 \end{array}$$

$$\begin{array}{r} 268 \\ -79 \\ \hline 17. \quad 189 \end{array}$$

$$\begin{array}{r} 641 \\ -57 \\ \hline 18. \quad 584 \end{array}$$

$$\begin{array}{r} 6,234 \\ -659 \\ \hline 19. \quad 5,575 \end{array}$$

$$\begin{array}{r} 4,662 \\ -787 \\ \hline 20. \quad 3,875 \end{array}$$



## Lesson 22

### Multiplication and Division

#### 22.1 Signs of Operation

multiplication asterisk \* ⠠⠠⠠⠠ (456,246)  
multiplication times cross × ⠠⠠⠠⠠ (46,246)  
multiplication times dot · ⠠⠠⠠⠠ (6,12456)  
divided by ÷ ⠠⠠⠠⠠ (46,34)  
slash / ⠠⠠⠠⠠ (456,34)  
begin displayed division ⠠⠠⠠⠠ (135)

#### 22.2 Signs of Comparison

is equal to = ⠠⠠⠠⠠ (123456)  
is greater than > ⠠⠠⠠⠠ (4,345)  
is less than < ⠠⠠⠠⠠ (4,126)  
is not equal to ≠ ⠠⠠⠠⠠ (46,123456)

#### 22.3 Division of Long Mathematical Expressions Between Braille Lines

Rules for dividing long expressions:

1. Start all expressions that are too long to fit on one line on a new line. Those that are short enough to fit on the line on which they start should be completed on that line.
2. Place all divided expressions within a notational phrase. A continuation indicator in these expressions makes it easy to realize that the expression is continued on more than one line. The notational phrase assures that the added line or lines are interpreted as notational information.
3. Expressions may be divided between lines in the following places, listed in order of preference: before a sign of comparison; before a sign of operation; before a fraction line; before baseline indicators; between factors that are enclosed in signs of grouping; after the last term of a modified expression; after the termination sign for a radical; and before any of the above that are within a superscript or subscript.
4. When an expression needs to extend over more than two lines, divide the expression such that each line after the first line begins with the same type of symbol whenever possible. For example, if an expression consists of two equal signs and requires three lines regardless of how it is divided, the second

and third lines should each begin with an equal sign if possible.

5. Should it not be possible to maintain such consistency, break long expressions as necessary to have the segments be approximately the same length. If one needs to be greatly longer than the others, try to have that be the first segment.
6. When multiple multi-line or single-line expressions follow each other directly, indent each runover segment by two spaces from the beginning of the first line of the expression to which it belongs. In a long expression surrounded by text, runover segments may be started directly under the beginning of the first line of the expression if so desired.

Example:

$$(235x+456+987y)-(123x+432+675y)+(26x-200-238y)$$

## 22.4 Linearly Displayed Multiplication

Simple multiplication problems can be presented linearly as the ones studied in the NUBS Literary Course. Keep in mind that with the notational phrase indicators, the whole mathematical statement is notational so we do not use additional notational indicators in the middle of the statement.

Examples:

$$3 \times 2 \times 5 = 30$$

$$(5a \times 3) + (6b \times 2) = 15a + 12b$$

## 22.5 Spatial Display Format for Multiplication

Often math problems are displayed in print to make it easier to handle large numbers. This type of problem is handled with Spatial Display in braille.

It is important to note that spatial format does not use notational or numeric indicators for spatial material that is entirely notational. We place an opening notational phrase indicator at the start of the blank line required before the displayed material. We also place a closing notational phrase indicator at the beginning of the blank line that follows the displayed material. Everything between these two indicators is notational.

The ruled line between a problem and its answer, separation line, is composed of a dot 5, followed by a series of dots 25, the same as used

in addition and subtraction.

### **22.5.1 Format issues for spatial display materials are as follows:**

1. A begin notational phrase indicator ¶¶ (56, 3456) must be placed at the start of the blank line required before a spatial display.
2. An end notational phrase indicator ¶¶ (56,3) must be placed at the start of the blank line required after a spatial display. If the last line of the example is on the last line of a page, the end notational phrase indicator may be placed at the end of that last line. A blank line should then be left at the top of the next page.
3. Entries must be aligned according to their decimal value or by their letters unless they have been intentionally misaligned as an exercise for the student.
4. The ruled line separation line must extend one cell beyond the left and right extremes of each example.
5. Operators and variables within a problem must be aligned.
6. When examples have identifier numbers or letters, they are placed on the same level as the ruled lines. One blank space must be left between the last symbol in the identifier and the symbol furthest left in the overall arrangement of the example including its separation lines. Since identifiers are inside the notational phrase, they do not need notational indicators. Optionally the identifiers may be provided their own notational indicators.
7. When spatial arrangements are placed side by side across a page without identifiers, at least two blank spaces must be left between every two separation lines.
8. When spatial arrangements are identified by number or letter and are placed side by side across a page, the ruled line of one spatial arrangement must be at least two cells distant from the identifier of the neighboring arrangement to its right.
9. In a spatial arrangement with multiplication, the multiplier and multiplicand must be aligned as in print. Dollar signs, commas, and decimal points should be placed as in print. The multiplication sign, if shown in print, is placed immediately before the multiplier. The separation line must extend one cell to the left and the right of the longest line above or below it. Additional separation lines must be the same length.
10. If the multiplication contains fractions, mixed numbers, or polynomials, its terms and indicators must be aligned vertically.

11. When a multiplication answer contains a comma or decimal point, the cells in the partial products above it should be left blank.
12. To show carried numbers, a carried number line, as used in addition, may be used to separate the carried numbers from the multiplicand; it consists of dots (56) followed by a line of dots (3456).
13. If the whole of an example cannot fit on the page on which it begins, it should be moved to the next page.

**Examples of structured format in multiplication problems:**

	101	⠠⠠⠠
x	22	⠠⠠⠠⠠⠠
	202	⠠⠠⠠⠠⠠⠠
	202	⠠⠠⠠⠠⠠
	2222	⠠⠠⠠⠠⠠⠠⠠⠠

Note that in braille the multiplication sign is positioned unspaced immediately before the multiplier.

Example: (showing column separation to keep digits aligned in the presence of decimal points.)

	345.7	⠠⠠⠠⠠⠠	
22.	x	2.77	⠠⠠⠠⠠⠠⠠⠠
	24.199	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	
	241.99	⠠⠠⠠⠠⠠⠠⠠⠠	
	691.4	⠠⠠⠠⠠⠠⠠⠠⠠	
	957.589	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	

In this example, we know that the answer will have 3 digits after the decimal point, because the two multipliers have a combined total of 3 digits after their decimal points.

Notice the alignment of the identifier number (22.) with the top ruled line and the positioning of the problem so its left-most element (the ruled line in this case) is one cell to the right of the identifier number. The identifier does not need a numeric indicator because it is within the notational phrase; it may have a numeric indicator if the transcriber prefers to add it.

### Drill #4

Braille the following. Show carried numbers in number 6 as an example of how carried numbers in multiplication might look.

1.  $12 \times 3 \times 10 = 360$     2.  $25 \cdot 3 = ?$     3. True or false.  $33 \cdot 3 < 100$

4. 
$$\begin{array}{r} 326 \\ \times 35 \\ \hline 1630 \\ 978 \\ \hline 11,410 \end{array}$$

5. 
$$\begin{array}{r} 189 \\ \times 31 \\ \hline 189 \\ 567 \\ \hline 5,859 \end{array}$$

6. 
$$\begin{array}{r} 746 \\ \times 212 \\ \hline 1492 \\ 746 \\ \hline 1492 \\ \hline 158,152 \end{array}$$

7. 
$$\begin{array}{r} 2.38 \\ \times 1.7 \\ \hline 1666 \\ 238 \\ \hline 4.046 \end{array}$$

8. 
$$\begin{array}{r} 78.2 \\ \times 65.3 \\ \hline 2346 \\ 3910 \\ \hline 4692 \\ \hline 5106.46 \end{array}$$

9. 
$$\begin{array}{r} 2837 \\ \times 416 \\ \hline 17022 \\ 2837 \\ \hline 11348 \\ \hline 1,180,192 \end{array}$$

## 22.6 Linearly Displayed Division

Some division problems are written out in a linear format. These will be done easily with an equation written out on a line. Here are the division symbols for your review.

divided by  $\div$  (46,34)

slash  $/$  (456,34)

begin displayed division  $\div$  (135)

When an identifier is placed one space before an equation, it optionally may be included within the notational phrase if desired for the equation; if included, the open notational phrase indicator is placed before the identifier.



that a remainder follows. A numeric indicator followed by the remainder is placed after the "R". The numeric indicator assures that the following number is not mistaken as a subscript.

10. Division problems should be started on a new page if they cannot fit on the page on which they began. If there is ever a problem that cannot fit on a whole page, it may be divided immediately before any of the ruled lines.

Look over the division problem below.

		⠠⠠⠠	
	121	R34	
2 . 39)	4753		
	39		
	85		
	78		
	73		
	39		
	34		
			⠠⠠⠠

1. The divisor (39) and the dividend (4753) are separated by ⠠⠠⠠ (dots 135), and the identifier number (2) is placed on level with the divisor and is followed by one blank cell.
2. The structure maintains columns of equal place value.
3. The remainder, if any, is placed above the ruled line and follows the quotient and one blank cell. Notice that the R34 that represents the remainder in this problem must include a numeric indicator to show that the 34 is not a subscript on R.
4. The ruled line begins above the division symbol, ⠠⠠⠠ dots (135) and extends one cell beyond the extreme right of the computation.

**Drill #5**

Braille the following examples to practice the rules for division in NUBS.

1.  $1755 \div 15 = 117$
2. What is  $21 \div 7$ ?
3.  $112 \div 7 = ?$

4.  $985 \div ? = 197$

5.  $? \div 21 = 13$

6.  $36 \times 4 \div 6 = 24$

$$\begin{array}{r} 19 \\ 23 \overline{) 437} \\ \underline{23} \\ 207 \\ \underline{207} \\ 0 \end{array}$$

$$\begin{array}{r} 32 \\ 18 \overline{) 576} \\ \underline{54} \\ 36 \\ \underline{36} \\ 0 \end{array}$$

$$\begin{array}{r} 8 \text{ R}4 \\ 79 \overline{) 636} \\ \underline{632} \\ 4 \end{array}$$

$$\begin{array}{r} 7.47 \\ 65 \overline{) 485.55} \\ \underline{455} \\ 305 \\ \underline{260} \\ 455 \\ \underline{455} \\ 0 \end{array}$$

$$\begin{array}{r} 13.7 \\ 2.4 \overline{) 32.88} \\ \underline{24} \\ 88 \\ \underline{72} \\ 168 \\ \underline{168} \\ 0 \end{array}$$

$$\begin{array}{r} 193 \text{ R} 15 \\ 64 \overline{) 12367} \\ \underline{64} \\ 596 \\ \underline{576} \\ 207 \\ \underline{192} \\ 15 \end{array}$$

### Exercises

1.  $12 \times ? = 144$

2.  $51,858 \div ? = 402$

3.  $3a \times 4b = 12ab$

4.  $ab \times cd = bd$

5.  $15z \div 3z = 5$

$$\begin{array}{r} 345 \\ \times 22 \\ \hline \end{array}$$

$$\begin{array}{r} 289 \\ \times 137 \\ \hline \end{array}$$

$$\begin{array}{r} 921 \\ \times 38 \\ \hline \end{array}$$

$$\begin{array}{r} 46.3 \\ \times 5.69 \\ \hline \end{array}$$

$$\begin{array}{r} 197 \\ \times 76 \\ \hline 1182 \\ 13,79 \\ \hline 14,972 \end{array}$$

11.  $45 \overline{) 9238}$

12.  $44 \overline{) 42271}$

13.  $36 \overline{) 178}$

$$\begin{array}{r} 11 \text{ R}44 \\ 63 \overline{) 737} \\ \underline{63} \\ 107 \\ \underline{63} \\ 44 \end{array}$$

$$\begin{array}{r} 2.68 \\ 46 \overline{) 123.28} \\ \underline{92} \\ 312 \\ \underline{276} \\ 368 \\ \underline{368} \\ 0 \end{array}$$

# Lesson 23

## Fractions

### 23.1 In-Line Fractions (Fractions printed on the same level)

When a slash occurs between numbers or letters all printed **on the same level**, it may be a date or an in-line fraction. Such a word is notational, and the standard two-cell slash ⠆⠆ (456,34) is used.

#### Examples:

3/4 lb of butter

⠆⠆⠆⠆⠆ ⠆⠆ ⠆⠆ ⠆⠆⠆⠆⠆

He wrote 15/30 on the list.

⠆⠆⠆ ⠆⠆⠆⠆⠆⠆ ⠆⠆⠆⠆⠆⠆⠆⠆ ⠆⠆ ⠆⠆ ⠆⠆⠆⠆⠆

12/25 is Christmas Day.

⠆⠆⠆⠆⠆⠆⠆⠆ ⠆⠆⠆ ⠆⠆⠆⠆⠆⠆⠆⠆⠆⠆⠆⠆⠆⠆⠆ ⠆⠆⠆⠆⠆

### 23.2 Simple Fractions

Fractions that show the numerator and denominator on different levels in print such as  $\frac{1}{3}$ , may have a slanted fraction bar or a horizontal bar in print. Both of these are treated notationally and require some additional symbols as shown below. The fraction with the slanted bar uses the two-cell fraction bar and the one with the horizontal bar uses the standard fraction bar. These forms of a fraction, where the numerator and denominator are on different levels in print, are both brailled using the begin and end fraction indicators.

### 23.3 Fraction Indicators

Begin fraction ⠆⠆ (1456)

Standard fraction bar ⠆⠆ (34)

Two-cell slash fraction bar ⠆⠆⠆ (456,34)

End fraction ⠆⠆ (3456)

**Examples:**

$$\frac{1}{3} \quad \frac{m}{h} \quad \frac{11}{16}$$

Notice that when the first element of the expression is a begin-fraction indicator, a notational indicator (56), rather than the numeric indicator (3456), is required to establish that the word is notational.

**23.4 Mixed Numbers**

All mixed numbers are brailled using the same format, even if the fractional part of the number is printed all on one level. A mixed number beginning with a digit uses the numeric indicator to set the notational mode; a mixed number beginning with a variable letter uses the notational indicator. When a diagonal slash is used in print, the two-cell slash is used for the fractional part of the mixed number.

**Examples:**

$$1 \frac{7}{8} \quad 4 \frac{1}{2} \quad x \frac{a}{b} \quad 33 \frac{1}{3}$$

For those who have experience with Nemeth code, you will find much of work with fractions familiar.

**23.5 Simple Fractions in Spatial Format**

As with the spatial display examples in the previous volumes, one or more spatial examples will be preceded by a notational phrase indicator placed at the beginning of the blank line required before the spatial material and at the beginning of the blank line that follows the spatial material. There is no need for a notational indicator at the start of a spatial fraction. There is, however, a need for a numeric indicator if a numerator or denominator does not contain any dots (1 or 4) and a notational indicator if a numerator or denominator does not contain any dots (3 or 6). When either dots 1 and 4 or dots 3 and 6 in a numerator or denominator are missing, a braille reader will have difficulty determining the position of the dots without an indicator. This means that a string of digits without

any characters that contain a dot 1 or dot 4 will need a numeric indicator, and a group of letters from a to j without other characters that contain a dot 3 or a dot 6 will need a notational indicator.

The fraction bar is formed by the open fraction indicator followed by a series of dots 25 and then followed by the close fraction indicator. The series of dots will be just long enough for the longer of the numerator or denominator to fit between the beginning and ending indicators. Both numerator and denominator, even if they are of different lengths, will begin at the left end of the dots 25.

The braille below could be shown together in one phrase if no print material was mixed in with them.

Examples:

$$\frac{1}{5} \quad \frac{3}{4} \quad \frac{c}{d} \quad \frac{x+y}{m}$$

Note that the last previous example does not need any notational indicator because a dot 1 or a dot 4 and a dot 3 or a dot 6 is present in both numerator and denominator.

$$\frac{65}{5} = 13$$

Note that with numbers, if there is more than one digit in a numerator or denominator, the indicator before the first digit covers the need for positioning for all the digits.

### 23.6 Mixed Numbers in Spatial Format

Mixed number fractions are shown with the whole number part on the same line as the fraction bar. Neither a numeric nor a notational indicator is used before the

whole number part of the mixed number placed above and below the problem or group of problems. Also, the open fraction indicator provides positioning information for the whole number part; indicators are still used in the numerator or denominator if they are composed only of digits or only of letters from a to j. In spatial format, neither the standard bar nor the slash bar is used.

Examples:

$$\begin{array}{ccc}
 \begin{array}{c} \dots \\ \dots \\ 3 \frac{5}{8} \\ \dots \\ \dots \\ \dots \end{array} &
 \begin{array}{c} \dots \\ \dots \\ 81^{15/16} \\ \dots \\ \dots \\ \dots \end{array} &
 \begin{array}{c} \dots \\ \dots \\ a \frac{d}{x} \\ \dots \\ \dots \\ \dots \end{array}
 \end{array}$$

### Drill #6

Practice with a few fractions here. Keep in mind that notational phrase indicators are required for equations.

$$\begin{array}{llll}
 1. \frac{3}{4} & 2. \frac{a+b}{c+d} & 3. \frac{1}{4} + \frac{3}{4} - \frac{1}{2} = \frac{1}{2} & 4. \frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd} \\
 5. \left( \frac{3}{2}a + \frac{1}{2}b \right) & 6. \$3.00 \times \frac{1}{5} = \$.60 & 7. \frac{100}{1000} = \frac{1}{10} &
 \end{array}$$

Practice brailleing the following examples in spatial format.

$$\begin{array}{llll}
 8. \frac{3}{8} & 9. \frac{9}{16} & 10. \frac{x+y}{z} & 11. \frac{a}{b+c} \\
 12. 26 \frac{7}{16} & 13. 7 \frac{11}{5} & 14. \frac{2x+4y}{6z} & 15. 12y \frac{j}{k}
 \end{array}$$

## 23.7 Cancellation in Fractions

Although subtraction schemes do not require cancellation indicators in NUBS, there are some situations where a technique to indicate cancellation is needed. Cancellation is used in the process of reducing fractions to lowest terms, and in the multiplication or division of fractions. In algebra, cancellation is used for the same reasons; however, there we are dealing with number-and-letter combinations rather than just with numbers in arithmetic. NUBS offers a mechanism for showing cancellation in which the extent of the cancellation is precise and the cancelled material remains readable. The cancellation indicators are:

Begin cancellation  $\ddot{::}$  (12346)

End cancellation  $\ddot{:}$  (246)

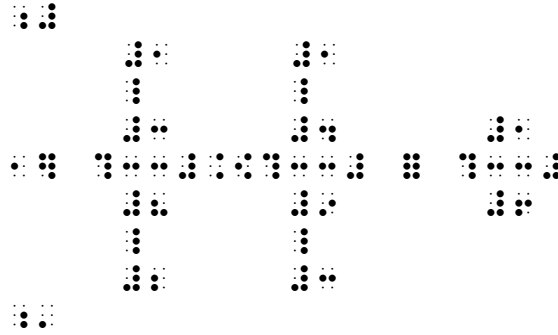
One-character cancellation  $\dot{:}$  (456)

### 23.7.1 Rules for Cancellation

1. Text in which cancellation occurs must be presented in spatial format.
2. Text containing cancellation is spatial and must be presented within a notational phrase. The notational phrase indicators will be placed at the start of the blank lines required by spatial display.
3. If the text to be cancelled occupies just one character, the one-character cancellation indicator must be placed above or below it, as appropriate. If the text to be cancelled occupies more than one cell, the begin-cancellation indicator must be placed above or below the first cell of the text and the end-cancellation indicator will be placed above or below the last cell of the text to be cancelled.

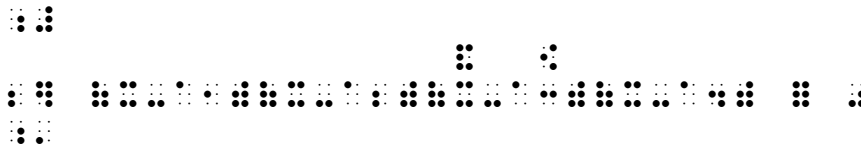
Example:

$$1. \quad \frac{\cancel{3}^1 \cancel{4}^1}{\cancel{8}^4 \cancel{9}^3} = \frac{1}{2 \cdot 3 \cdot 6}$$



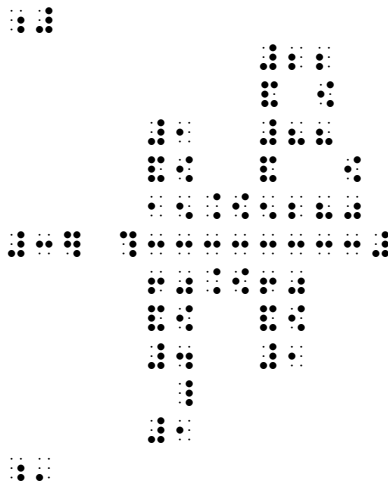
This example shows how two fractions are multiplied. The 3 in the first numerator cancels with the 9 in the second denominator. These cancelled numbers are replaced respectively by 1 and 3. The number 8 in the first denominator and the 4 in the second numerator have a common factor of 4. These numbers are cancelled and replaced by the result obtained by dividing each by 4. The end result is obtained by multiplying the two new numerators and the two new denominators. 1 by 1 and 2 by 3 giving the fraction  $\frac{1}{6}$ .

$$2. \quad (x-a_1)(x-a_2)(x-a_3)(x-a_4) = 0$$



Example 2 shows the product of four algebraic expressions in which the third factor has been cancelled without a replacement.

$$3. \quad \frac{\begin{array}{r} 22 \\ 1 \ 88 \\ \hline 15 \times 5280 \\ 60 \times 60 \\ \hline \cancel{4}^1 \ 1 \\ 1 \end{array}}{1}$$



Example 3 above shows cancellation within a fraction that needed two cancellations in a numerator and two in a denominator.

### 23.8 Complex fractions

A complex fraction is one that has a fraction whose numerator or denominator (or both) either are fractions, or contain fractions. We recall that an in-line fraction, consisting of a number followed by a slash and another number all on the same level, is not treated as a fraction. Thus, if both numerator and denominator of what looks like a complex fraction are in-line fractions, then it is actually a simple fraction and does not need the complex fraction symbols.

#### Example of an in-line fraction within a fraction:

$$\frac{2/3}{3/2} \quad \text{⠠⠨⠆⠠⠨⠃⠠⠨⠃⠠⠨⠆}$$

#### 23.8.1 Symbols used in complex fractions:

Open complex fraction ⠠⠨⠆ (6,1456)

Complex fraction line ⠠⠨⠆ (6,34)

Close complex fraction ⠠⠨⠆ (6,3456)

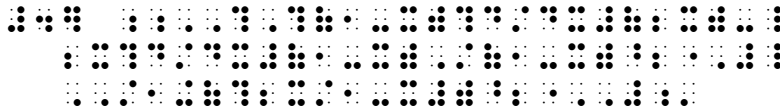
The dot 6 used in these symbols is referred to as a complexity indicator. A simple fraction does not use this indicator. A complex fraction uses just one indicator and is also referred to as a first order complex fraction. As fractions increase in complexity, more indicators will be used to signify their level.

Fractions that make up a numerator or denominator are brailled as simple fractions. A complex fraction in NUBS begins with a notational indicator followed by the **open complex fraction** symbol. The numerator is entered as a simple fraction or character. The **complex fraction line** symbol is entered followed by the denominator entered as a simple fraction or character. The **close complex fraction** symbol is entered to complete the complex fraction.





$$4. \frac{(1-x) \frac{d}{dx}(2x) - 2x \frac{d}{dx}(1-x)}{(1-x)^2} \frac{1}{1 + \left(\frac{2x}{1-x}\right)^2}$$



Note the need to return to baseline after the superscripts. Note the use of a notational phrase for the divided example in number 4.

### 23.10 Higher-Order Fractions and Spatial Format

The more complex a fraction becomes, the more difficult it is to read in the linear form. Consider the optional braille versions of these hypercomplex fractions which can provide a more readable format. There are two other basic options for displaying the example above. These are either all spatial or part spatial and part linear. In the spatial format example, identifying numbers are placed at the start of the line for the major fraction level.

Let's examine these issues for each of the above examples.

Example 2 – all spatial

$$2. \frac{\frac{1}{x}}{y+a} \frac{1}{b}$$

The easiest way to build this example is as follows:

- 1) Create the simple fraction as an independent expression, starting near the middle of a braille line. Be sure to left adjust the denominator.

- 2) Under that denominator, create the intermediate level fraction bar, starting it to extend one cell before and one cell beyond the fraction bar above.
- 3) Place the denominator for that level, left adjusted, below that fraction bar.
- 4) On the next line, start with the identifying number and then create the major level fraction bar, extending it one cell before and one cell after the previous fraction bar.
- 5) Place the major denominator, left adjusted, below that fraction bar.
- 6) Reposition the entire display to its proper placement on the page.

The format above shows the actual physical structure of the print text (except for left adjusted denominators). A less cumbersome option is a mix of spatial and linear formats as follows:

Example  $\frac{\frac{1}{x}}{y+a}$   $\frac{1}{x}$   $y+a$   $b$

**The format below treats everything above the major fraction bar as a complex fraction** using the linear format. This is often the preferred format in that it takes up fewer braille lines and is easier to read. Note that the major denominator is left-adjusted under the brailled fraction line. This use of spatial format for fractions provides a structure that is compact (only 3 braille lines) and yet is easy to read.

Example 3  $\frac{\frac{1}{\frac{4}{3}}}{\frac{1}{5}}$   $\frac{1}{4}$   $\frac{3}{1}$   $\frac{1}{5}$   $5$

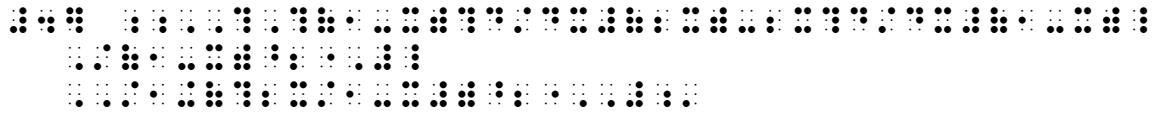
Here again, this format is easier to read and involves only a complex fraction as the numerator in a displayed format and a single digit as the denominator. Again, notice the required notational indicator or a numeric indicator preceding the denominators; each, standing alone, are missing either an upper dot, 1 or 4, or a lower dot, 3 or 6. The indicators make it easier to know whether a digit or a letter is there.

Another form provides a separate braille line for the beginning of each fraction level. This representation is entirely linear, difficult to read, but requires only three braille lines. Because it is on separate lines, it must be placed within a

notational phrase.

Example 4 
$$\frac{(1-x)\frac{d}{dx}(2x) - 2x\frac{d}{dx}(1-x)}{(1-x)^2}$$
  

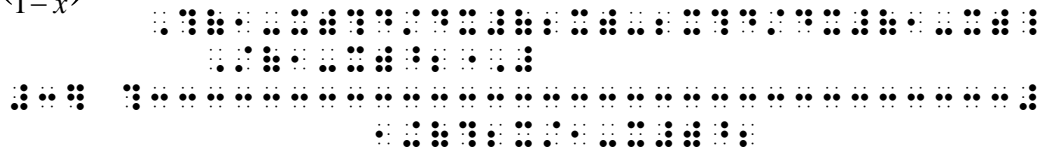
$$1 + \left(\frac{2x}{1-x}\right)^2$$



Yet another representation is a “first level” spatial – using only the major fraction structure base. Both the numerator and the denominator of the major fraction are shown linearly.

$$\frac{(1-x)\frac{d}{dx}(2x) - 2x\frac{d}{dx}(1-x)}{(1-x)^2}$$

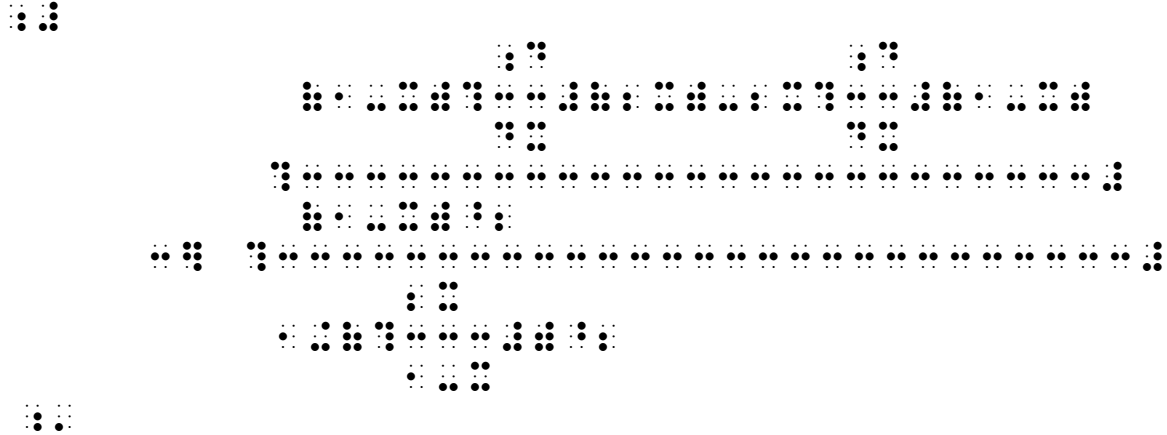
$$1 + \left(\frac{2x}{1-x}\right)^2$$



If one additional “layer” of spatial structure is used, the following format results.

$$\frac{(1-x)\frac{d}{dx}(2x) - 2x\frac{d}{dx}(1-x)}{(1-x)^2}$$

$$1 + \left(\frac{2x}{1-x}\right)^2$$



This is probably the most readable, especially for blind students in their first exposure to such complex fractions. It does require five additional braille lines, but readers may find that is a fair trade in the learning process.

### Drill #7

Practice braille complex fractions, numbers 1-6, in both the linear and the spatial forms. Use spatial for number 7.

1.  $\frac{1}{\frac{4}{3}}$     2.  $\frac{\frac{3-1}{4+3}}{\frac{2-1}{3+5}}$     3.  $\frac{2\frac{1}{3}}{4\frac{1}{5}}$     4.  $\left(\frac{3}{2}\right) \times \left(\frac{1}{2}\right) \times \left(-\frac{1}{2}\right)$     5.  $\frac{\frac{x-3}{x+1}}{\frac{x}{y}}$     6.  $\frac{a + \frac{2}{\alpha+1}}{a - \frac{3}{\alpha-2}}$

7.  $\cancel{(x-1)} \frac{3x}{\cancel{(x-1)}} + \cancel{(x-1)} \frac{6x-9}{\cancel{(x-1)}} = (x-1)9$

### Exercises

1.  $\frac{1}{6} \times \frac{4}{1} = \frac{4}{6} = \frac{2}{3}$     2.  $\frac{2}{5} \times \frac{3}{2} = \frac{6}{10} = \frac{3}{5}$     3.  $\frac{15}{34} \times \frac{17}{5} = \frac{3}{2} = 1\frac{1}{2}$     4.  $\frac{5}{9} \times \frac{8}{25} = \frac{1}{9} \times \frac{8}{25} = \frac{8}{225}$

$$5. \frac{36}{4} \times \frac{16}{6} = \frac{24}{1} = 24 \quad 6. \frac{3}{10} + \frac{2}{5} = \frac{3}{10} + \frac{2 \times 2}{5 \times 2} = \frac{3}{10} + \frac{4}{10} = \frac{7}{10} \quad 7. \frac{3}{4} - \frac{2}{5} = \frac{3 \times 5}{4 \times 5} - \frac{2 \times 4}{5 \times 4} = \frac{15}{20} - \frac{8}{20} = \frac{7}{20}$$

$$8. \frac{\frac{1}{2}}{\frac{5}{3}} \quad 9. \frac{\frac{11}{12}}{\frac{5}{12}} \quad 10. \frac{\frac{3}{10}}{\frac{1}{2}} \quad 11. \frac{1\frac{1}{3}}{\frac{9}{10}} \quad 12. \frac{\frac{1}{4}}{\frac{3}{4}} \quad 13. \frac{\frac{1}{2} + \frac{1}{3}}{\frac{7}{9}} \quad 14. \frac{\frac{3-1}{4+3}}{\frac{2-1}{3+5}}$$

$$15. \frac{\frac{1}{3} + \frac{1}{4}}{\frac{4}{5} - \frac{1}{2}} \quad 16. \frac{2\frac{1}{3}}{4\frac{4}{5}} \quad 17. \frac{33\frac{1}{3}}{100} \quad 18. \frac{a}{\frac{b}{c}} \quad 19. \frac{X+2+\frac{2}{x+5}}{x+6+\frac{6}{x+1}}$$

$$20. \frac{\frac{a^2-6a+5}{a^2+13a+42}}{\frac{a^2-4a+3}{a^2+3a-18}}$$



After the notational phrase indicator in the second example, we show the "a", followed by the superscript indicator. After the exponent 2, we must use the baseline indicator to show that we are returning to the baseline and that the +b<sup>2</sup> is not part of the exponent on "a". The b<sup>2</sup> is returned to the baseline by the space that follows it. There is also a dot 5 after the c<sup>2</sup> before the closing notational phrase indicator. The notational phrase indicators maintain the notational mode through the spaces around the equal sign.

Look at the next two expressions to see the significance of the **baseline indicator**. The exponent on "a" is different in each example.

$$a^{2x}+3+4b^3 \quad \text{Exponent on } a \text{ is } 2x.$$

$$a^{2x+3}+4b^3 \quad \text{Exponent on } a \text{ is } 2x+3.$$

The exponent consists of everything that is after the superscript indicator and before either a baseline indicator or a space.

### 24.1.3 Numeric Subscripts

NUBS provides an abbreviated notation for numeric subscripts. This method dispenses with the subscript indicator, and the reader is returned to the base level when the number has ended. This notational abbreviation is available only when a number is a subscript to a letter; it does not apply to numbers which are subscript to other symbols. The letter to which a number is a subscript may be in any case, and in any font, or from any alphabet. However, the letter to which the number is a subscript must be at the base level.

Examples. (of numeric subscripts)

$$1. \quad x_1+x_2+x_3$$

$$2. \quad a_{11}x_1+a_{12}x_2+ \dots +a_{1n}x_n = b_1$$

$$\dots +a_{2n}x_n = b_2$$

$$3. \quad a_{n1}x_1+a_{n2}x_2+ \dots +a_{nn}x_n = b_n$$

$$\dots +a_{m1}x_1+a_{m2}x_2+ \dots +a_{mn}x_n = b_m$$







3.  $y = 9 + x^4 - 6x^3 + 18x + 3x^2$
4.  $\text{NH}_4$  is ammonium.
5.  $\text{C}_6\text{H}_{12}\text{O}_6$  is glucose.
6.  $\text{C}_{12}\text{H}_{22}\text{O}_{11}$  is sucrose.
7. Evaluate:  $\frac{2 \cdot 8^2 - 2^2 \cdot 8}{2 \cdot 8}$  (Use spatial display)
8. Copyright © 2008 by The McGraw-Hill Companies
9. Evaluate if  $r = 2$ ,  $s = 3$ , and  $t = 11$ ,  $s^2 + (r^3 - 8) + 5$ ,  $3r(r+s)^2 - 1$ , and  $t^2 + 8st + r^2$

## Exercises

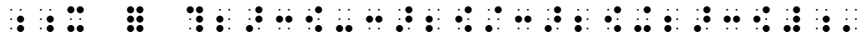
1.  $ax_2$
2.  $A_{56}$
3.  $X_{5.3}$
4.  $\text{CO}_3$
5.  $\text{Na}_3$
6.  $\text{TE}_{12}$
7.  $y_{-2}$
8.  $2m^3 + 2m^3$
9.  $4(x-y)^3 - 2(x-y)^3$
10.  $3a^3 + 6a^6b^2 + 9a^9b^3$
11.  $(x_1y_1 + x_2y_2)$
12.  $8x^2 + 8x - 15$
13.  $([\text{CH}_3]_2\text{CH})$
14.  $12\text{ft}^2 \times 12\text{ft}^2 = 144\text{ft}^2$
15.  $6x^{2a} + 11x^a + 4$
16.  $y_n + y_{n-1} = z$

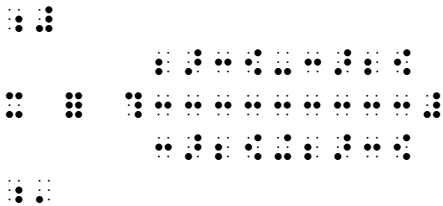


$$\frac{2\sqrt{3}-3\sqrt{2}}{3\sqrt{2}+2\sqrt{3}}$$

Fractions and radicals are used together in many math statements. They may be handled either in a linear format or in a spatial format. The example below shows both ways of brailleing such an example.

$$x = \frac{2\sqrt{3}-3\sqrt{2}}{3\sqrt{2}+2\sqrt{3}}$$

Linear: 

Spatial: 

Although the linear form takes fewer lines, the spatial form makes it much easier for young readers to follow the structure of the expression.

### 25.1.2 Indexed and Nested Radicals

Radicals can sometimes have a more complex structure than those just shown, such as:

- 1) Indexed radical: displaying an index such as a 2 for square root or a 3 for cubed root
- 2) Nested radicals: having a radical occur inside another radical

## 25.2 Indexed Radicals

A radical is used to determine the **root** of a number or of an expression. In the radicals we have studied so far, that root is assumed to be 2 when no other





- b) Zero or more ascender elements (456) ⠆ bridge the space above the begin-radical indicator to the vinculum (to the dot (56) cell). The order of the radical determines how many ascender cells are needed (to elevate the vinculum so that internal subordinate vinculum have sufficient vertical space).
- 4) Dot (6) (to indicate order) is not used in spatial representations of radicals.
- 5) No end-radical indicator is needed because the end of the vinculum shows the extent of the radicand.

**Examples:**

Note the beginning and ending notational phrase indicators placed before and after the braille section of the display below. These have been placed to simulate a page where there were not any print examples.

⠆ ⠆

$\sqrt{x + y}$	⠆ ⠆ ⠆ ⠆ ⠆ ⠆ ⠆ ⠆
$\sqrt{x + \sqrt{x + y} + y}$	⠆ ⠆
$\sqrt[3]{x^2 + \sqrt[3]{x^2 + y^2} + y^2}$	⠆ ⠆

⠆ ⠆



4. Simplify: a.  $\frac{4\sqrt{6}-3\sqrt{2}}{7\sqrt{3}+2\sqrt{5}}$       b.  $\frac{2-\sqrt{\frac{1}{4}}}{3-\sqrt{\frac{1}{2}}}$

5. Find:  $\sqrt{(30)} \times \sqrt{(10)} \times \sqrt{(10)}$

6. Find: a.  $\sqrt[3]{16} + \sqrt[4]{162}$     b.  $\sqrt[5]{8} \cdot \sqrt[5]{2} \cdot \sqrt[5]{2}$     c.  $\sqrt[4]{8r^2s^3t^4} \sqrt[4]{r^6st^2}$

7. Express each of the following as an integer or as a single radical.

7. a.  $\sqrt{\sqrt{a}}$       b.  $\sqrt[3]{\sqrt{729}}$       c.  $\sqrt{\sqrt[3]{\sqrt[4]{b^2}}}$

### Exercises

Provide answers in both linear and display formats.

1.  $\sqrt{63}$
2.  $\sqrt{a+b}$
3.  $\sqrt{a^2+b^2}$
4.  $\sqrt[3]{\frac{a+b}{a-b}}$
5.  $(r^2\sqrt{r^3})^3$
6.  $\sqrt{\sqrt{a^2+b^2}}$
7.  $\sqrt{80a^6b^2}$
8.  $\sqrt{48x^3y^3}$
9.  $\frac{4\sqrt{6}-3\sqrt{2}}{7\sqrt{3}+2\sqrt{5}}$
10.  $\sqrt{\sqrt{15} + \sqrt{17} + \sqrt{117}}$
11.  $\sqrt[3]{\sqrt{729}}$
12.  $\sqrt{s^3\sqrt{s\sqrt{s}}}$
13.  $\sqrt[3]{\sqrt[3]{\sqrt{256}}}$
14.  $\sqrt{1-\sqrt{a-b}} \times \sqrt{1+\sqrt{a-b}}$
15.  $\sqrt{\sqrt[3]{16}} = \sqrt[3]{\sqrt{16}}$



# Lesson 26

## Geometry Shapes and Modifiers

### 26.1 In General

This chapter on Geometry and Trigonometry, provides the symbols needed for handling geometry and trigonometry as well as necessary rules for correct use of those symbols within a transcription.

#### 26.1.1 Geometry

Geometry is the study of the properties and relationships involving lines and planes, etc. It is divided into two basic areas – plane geometry, which deals with two-dimensional figures such as lines and surfaces -- and solid geometry which deals with three dimensional figures such as spheres and cubes.

### 26.2 Shapes in Geometry

In the study of mathematics, especially in the area of geometry, the representation of shapes is important. Key shapes and their braille form are shown in the table on the following page. This is a complete list except that others may be devised using the basic guidelines shown here; transcriber's notes would be used to explain such additions. These shapes are all notational and each requires a notational indicator.

Shapes are brailled using three braille cells; the first two form the **compound prefix** and the third is the **root** which identifies the particular shape.

In the braille column of the table below, note the compound prefix in the first two cells ⠠⠠ (456,5). Note the third cell in the Braille column, the root, formed by a letter or symbol. The three cells together stand for the shape and would be preceded by a notational indicator if it was not already within a notational word or phrase.

Shape	Print	Braille	Dots
Angle	∠	⠠⠠⠠	(456,5,2346)
Arc - concave	⤿	⠠⠠⠠	(456,5,1)
Arc - convex	⤿	⠠⠠⠠	(456,5,3)
Circle (ring)	◯	⠠⠠⠠	(456,5,14)
Diamond	◊	⠠⠠⠠	(456,5,145)
Ellipse	⬭	⠠⠠⠠	(456,5,15)
Equilateral triangle	△	⠠⠠⠠	(456,5,25)
Hexagon	⬡	⠠⠠⠠	(456,5,235)
Is parallel	∥	⠠⠠⠠	(456,5,123)
Is perpendicular	⊥	⠠⠠⠠	(456,5,1234)
Octagon	⬢	⠠⠠⠠	(456,5,236)
Oval	◌	⠠⠠⠠	(456,5,135)
Parallelogram	▭	⠠⠠⠠	(456,5,1245)
Pentagon	⬠	⠠⠠⠠	(456,5,26)
Quadrilateral	▭	⠠⠠⠠	(456,5,12345)
Rectangle	▭	⠠⠠⠠	(456,5,1235)
Rhombus	◊	⠠⠠⠠	(456,5,125)
Right angle	⊥	⠠⠠⠠	(456,5,1456)
Right triangle	◡	⠠⠠⠠	(456,5,1236)

Square	□	⠠⠠⠠	(456,5,256)
Star	☆	⠠⠠⠠	(456,5,234)
Trapezoid	▭	⠠⠠⠠	(456,5,1356)
Triangle, generic	△	⠠⠠⠠	(456,5,2345)
Measure of angle	m∠	⠠⠠⠠⠠	(134,456,5,2346)

### 26.3 Shading of Shapes

A shape may be shown as hollow, shaded, or solid. The shapes shown in the table above are hollow. A solid shape will be shown with the prefix and then ⠠ (456) will be inserted before the root. A shaded shape will be shown with the prefix and then ⠡ (46) will be inserted before the root.

#### Examples:

solid rectangle ⠠⠠⠠⠠  (456,5,456,1235)  
shaded square ⠠⠠⠠⠠  (456,5,46,256)

Because all of the shapes are notational, the notational indicator is required in expressions such as those above. They will be added any time a shape is used unless the notational mode has already been established for the word or phrase of which the shape is a part.

An exception to the solid rule is the solid dot which is represented by the symbol for a notational period. Solid dot • ⠠ (12456)

#### 26.3.1 Geometric Shapes in Plane Geometry

Many of the shapes discussed above are used in the presentation of problems in geometry. When a parallelogram has vertices of M, N, O, and P, it is more convenient to refer to this as ▭ MNOP rather than labeling it as “parallelogram MNOP”.

These shape symbols can be used to identify geometric shapes by their vertices; the corners of a shape are usually identified by capital letters. In braille



The simplest of these to express in braille are those shapes that can be used as modifiers in the “**modification**” technique.

There are **two groups** of shapes that can be used as modifiers. These groups are “General Modifiers” and “Geometric Modifiers.” The elements of each of these groups are listed below.

### General Modifiers

Shape	Print	Braille	Dots
acute accent	´	⠠⠠	(4,35)
asterisk	*	⠠⠠	(456,246)
breve	˘	⠠⠠⠠	(456,5,1)
circumflex accent	ˆ	⠠⠠	(6,26)
diaeresis	¨	⠠⠠	(46,25)
dot	.	⠠	(12456)
grave accent	`	⠠⠠	(4,26)
hat	^	⠠⠠	(6,26)
left arrow	←	⠠⠠	(4,246)
macron (bar)	—	⠠	(156)
right arrow	→	⠠⠠	(4,135)
ring	◊	⠠⠠	(46,346)
tilde	~	⠠⠠	(46,35)
two-way arrow	↔	⠠⠠	(4,123456)
umlaut	¨	⠠⠠	(46,25)

The general modifiers may be used any place where print text uses them in both literary and mathematical materials.

## 26.5 Geometry Modifiers

Shape	Print	Braille	Dots
full line	$\longleftrightarrow$	$\begin{smallmatrix} \dots \\ \dots \end{smallmatrix}$	(4,123456)
half lines			
closed left	$\bullet\longrightarrow$	$\begin{smallmatrix} \dots \\ \dots \end{smallmatrix}$	(456,4,135)
closed right	$\longleftarrow\bullet$	$\begin{smallmatrix} \dots \\ \dots \end{smallmatrix}$	(456,4,246)
open left	$\circ\longrightarrow$	$\begin{smallmatrix} \dots \\ \dots \end{smallmatrix}$	(46,4,135)
open right	$\longleftarrow\circ$	$\begin{smallmatrix} \dots \\ \dots \end{smallmatrix}$	(46,4,246)
line segments			
left closed, right closed	$\bullet\text{---}\bullet$	$\begin{smallmatrix} \dots \\ \dots \end{smallmatrix}$	(456,456,156)
left closed, right open	$\bullet\text{---}\circ$	$\begin{smallmatrix} \dots \\ \dots \end{smallmatrix}$	(456,46,156)
left open, right closed	$\circ\text{---}\bullet$	$\begin{smallmatrix} \dots \\ \dots \end{smallmatrix}$	(46,456,156)
left open, right open	$\circ\text{---}\circ$	$\begin{smallmatrix} \dots \\ \dots \end{smallmatrix}$	(46,46,156)

### 26.5.1 Rules for Modification

Here are three modified expressions where the “over and under” material is contained in the list of “General Modifiers” shown above. The expressions are notational and a notional indicator must be used if that mode has not already been established.

$$\overrightarrow{xy} \quad \overset{\circ}{A} \quad \underline{MN}$$

The following steps are required to express these in braille.

1. Enter the notational indicator if that mode has not previously been established.
2. Enter the directly-over or directly-under indicator.  
For modification above use  $\begin{smallmatrix} \dots \\ \dots \end{smallmatrix}$  (126) (Think: gh – go high?)

For modification below use ⠠ (146) (Think: sh – sink here?)

3. Enter the term to be modified.
  4. Enter the modification symbol. There is no termination symbol; the modifier forms the end of the modified term.
- The expressions above would be brailled as:

$$\overrightarrow{xy} \quad \overset{\circ}{A} \quad \underline{MN}$$

**Additional Examples:** (using “general” modifiers from the first list )

1.  $\bar{x}$  ⠠⠠⠠⠠⠠ (bar over  $x$  -- notational indicator , if needed, modification above, letter x, bar)
2.  $\tilde{x}$  ⠠⠠⠠⠠⠠ (tilde over  $x$  )
3.  $3.\dot{1}\dot{4}\dot{2}\dot{8}\dot{5}\dot{7}$  ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠ (a repeating decimal)
4.  $\overline{\bar{x} + \bar{y}}$  ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

The conjugate of, the conjugate of  $x$  plus the conjugate of  $y$ . Notice how the  $\bar{x}$  and the  $\bar{y}$  are enclosed in an additional set of modification symbols. It is transcribed as notational indicator; begin modification above for the long top bar; then modification above for the  $x$  followed by the bar symbol to end the modification of  $x$ ; then the plus sign; then begin modification above for the  $y$  followed by the bar symbol to end the modification of  $y$ ; then the last bar symbol to end the long top bar modification.

5.  $\hat{y}$  ⠠⠠⠠⠠⠠ (hat over  $y$ )
6.  $\underline{x}$  ⠠⠠⠠⠠⠠ (right arrow under  $x$ )
7.  $\cong$  ⠠⠠⠠⠠⠠ (is congruent to)

## 26.6 Limits of the Modification Process

The “modification” process is used only when the element to be placed above or below a math expression is composed of symbols that are included in the lists of either the General Modifiers or the Geometry Modifiers. When it is

necessary to place other symbols over or under a math expression, a procedure that uses **overscripts** or **underscripts** is required. It is important to understand that **all material** that is shown as text above or below a principle expression but is not either a general modifier or a geometry modifier must use the “**overscript or underscore**” method described in Section 29.2.

## 26.7 Other Uses of Mathematical Shapes

Occasionally, an author will use a shape to represent a missing element in a mathematical expression. Usually this is presented in a form where the reader is asked to determine the meaning of the shape, from the context of the expression. Notice that the use of spaces depends on what function they serve. For signs of comparison, spaces will need to be left before and after them. A statement such as this will be placed within a notational phrase so that those spaces will not be delimiters and the notational mode will still be in effect.

For signs of operation, no spaces are present before or after an operator in print. However, in braille, since a blank cell is required both before and after the identifying letters of a braille shape, we must leave a space before and after the shape. We must keep in mind that in print, there would not be any space after the operator which the shape represents.

Examples:

- Write + or - for each shape.

$$5 \triangle 2 = 7$$

$$\text{a. } 5 \triangle 2 = 7$$

$$\text{or } 5 \triangle 2 = 7$$

$$\text{b. } 9 \square 1 = 8$$

$$\text{or } 9 \square 1 = 8$$

$$\text{c. } (a-b) \square a = -b$$

$$\text{or } (a-b) \square a = -b$$

- 1 qt =  $\square$  pt and 4 qt =  $\bigcirc$  pt.

$$1 \text{ qt} = \square \text{ pt and } 4 \text{ qt} = \bigcirc \text{ pt.}$$

or

⋮⋮⋮ ⋮⋮⋮ ⋮⋮ ⋮ ⋮⋮⋮ ⋮⋮ ⋮⋮⋮ ⋮ ⋮⋮ ⋮ ⋮⋮⋮ ⋮⋮⋮⋮⋮

### 26.8 Modifiers in Plane Geometry

The second list of modifiers is used to describe various forms of lines: full lines that extend infinitely, half lines that have an endpoint at one end and extend infinitely in the opposite direction, and line segments that are limited at both ends. Here are some examples of the modification process used in the geometric context.

**Examples:** (of geometric modifiers)

↔ ⋮⋮⋮⋮⋮⋮⋮ (full-line AB)  
AB

○→ ⋮⋮⋮⋮⋮⋮⋮⋮ (The half-line PQ excludes point P.)  
PQ

●—○ ⋮⋮⋮⋮⋮⋮⋮⋮ (The line segment CD in which C is included and D is excluded.)  
CD

△ABC ≅ △DEF

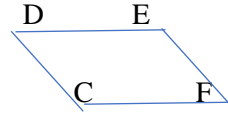
⋮⋮⋮⋮⋮⋮ ⋮⋮⋮⋮⋮⋮⋮ ⋮⋮⋮⋮⋮ ⋮⋮⋮ ⋮⋮⋮⋮⋮⋮⋮⋮  
(Triangle ABC is congruent to triangle DEF.)

In the case of half-lines and line segments as shown in print, a solid dot is used at the end of the modifier if the endpoint is included, and a hollow dot is used if the endpoint is excluded. In braille, each of these expressions must be preceded by a notational indicator if the notational mode has not previously been established.

### Drill #10

1. What is the area of □HIJK if each side is 8 inches long?

- What is the perimeter of  $\square ABCDEF$  if all sides are 2 feet long?
- $BC \perp DE$
- $MN \parallel OP$
- $\overline{OP} + \overline{QR} = \overline{OR}$
- $\triangle XYZ$  is isosceles.
- Find the area of  $\triangle ABC$  if the base  $BC = \overline{15}$  and the height equals 4.



- Find the perimeter of  $\square CDEF$  where  $CD = \overline{8}$  and  $DE = \overline{5}$ .
- $\triangle ABC$  is a right triangle with hypotenuse  $BC$ .
- Lines  $AB$  and  $CD$  intersect at  $P$ .

### Exercises

- $XY \parallel RS$
- $AB \perp CD$
- $7 \times \triangle = 4 \times \triangle$
- $\triangle ADM = \triangle BEM$
- $\angle ABD + \angle DBE = ?$
- $A \rightarrow B$
- $\overline{X} \times \overline{Y} = \overline{XY}$
- $\angle a + \angle b + \angle c = 180^\circ$
- Does  $\frac{ST}{UV}$  equal  $\frac{UV}{ST}$ ?
- $0.2367\overline{78}$        $7.13\overline{13} \dots$
- In  $\square ABCD$ ,  $BC = AD$ . If diagonals  $AC$  and  $BD$  intersect at  $E$ , prove that  $\triangle DEA \sim \triangle CEB$ .

-

## Lesson 27

# Trigonometry

### 27.1 Trigonometric Function Abbreviations

Trigonometry is characterized by the generous use of function abbreviations. It is customary for these function abbreviations to be printed in the host type. All of the trigonometric function abbreviations are listed below. The notational indicator is necessary for each abbreviation when it is used in a mathematical context, since they are notational and do not use contractions.

arccosecant	arccsc		inverse cosine	$\cos^{-1}$
arccosine	arccos		inverse cotangent	$\cot^{-1}$
arccotangent	arccot		inverse cotangent	$\text{ctn}^{-1}$
arccotangent	arcctn		inverse hyperbolic cosecant	$\text{csch}^{-1}$
arcsecant	arcsec		inverse hyperbolic cosine	$\text{cosh}^{-1}$
arcsine	arcsin		inverse hyperbolic cotangent	$\text{coth}^{-1}$
arctangent	arctan		inverse hyperbolic cotangent	$\text{ctnh}^{-1}$
cosecant	csc		inverse hyperbolic secant	$\text{sech}^{-1}$
cosine	cos		inverse hyperbolic sine	$\text{sinh}^{-1}$
cotangent	cot		inverse hyperbolic tangent	$\text{tanh}^{-1}$
cotangent	ctn		inverse secant	$\text{sec}^{-1}$
hyperbolic cosecant	csch		inverse sine	$\sin^{-1}$
hyperbolic cosine	cosh		inverse tangent	$\tan^{-1}$
hyperbolic cotangent	coth		radian	rad
hyperbolic cotangent	ctnh		secant	sec
hyperbolic secant	sech		tangent	tan
hyperbolic sine	sinh		hyperbolic tangent	tanh

There are other function abbreviations which are not specific to trigonometry. They will follow the same basic rules for notation. They will be provided in an appendix.



arc sin x (a space in the function name in print.)

arccos x (no space in the function name in print.)

$\sin^{-1} \phi = 1/\sin \phi$  ( $\Phi$  is Greek capital phi)

### Drill #11

1.  $\sin(x+y)$

2.  $\sin \pi/3$

3.  $\sin 30^\circ \cos 45^\circ$

4.  $\sin x \cos y$

5.  $2\sin x + 3\cos y$

6. Are sine and arcsine true functions?

7.  $2 \sin 45^\circ \cos 45^\circ = \sin 90^\circ$

8.  $\cos^2 x \sin^2 x$

9. Solve  $e^{\sin x}$  when  $x = 45^\circ$ .

10.  $\tan \theta = \frac{\sin \theta}{\cos \theta}$

11.  $\sin \frac{5\pi}{12}$

12.  $\cos 225^\circ = -\sqrt{\frac{1 + \cos 450^\circ}{2}}$

13.  $\cos 30^\circ = \frac{x}{r} = \frac{1}{2} \sqrt{3}$

14. Evaluate the expression  $\sin 30^\circ \cos 45^\circ$ .

15. Evaluate  $2\sin x + 3\cos y$  if  $x$  equals  $10^\circ$  and  $y$  equals  $35^\circ$ .

16. What is the meaning of  $\arccos \theta$ ?



## Lesson 28

### Determinants and Matrices

#### 28.1 Signs of Grouping

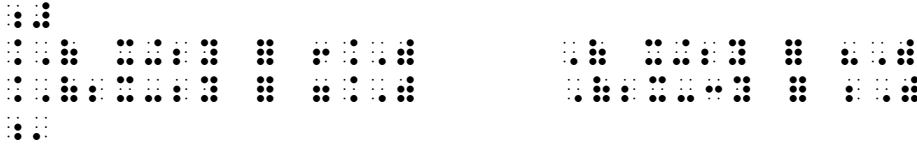
The list below gives grouping signs defined in NUBS that were not provided in the NUBS Literary Manual.

Shape	Print	Braille	Dots
left brace	{	⠠⠠⠠	(456,46,12356)
right brace	}	⠠⠠⠠	(456,46,23456)
left bracket	[	⠠⠠⠠	(456,4,12356)
right bracket	]	⠠⠠⠠	(456,4,23456)
left boldface bracket	[	⠠⠠⠠	(4,456,12356)
right boldface bracket	]	⠠⠠⠠	(4,456,23456)
vertical bar left and right		⠠⠠⠠	(4,456,1256)
double vertical bars left and right		⠠⠠⠠	(4,4,1256)
extended left brace		⠠⠠⠠	(46,6,12356)
extended right brace		⠠⠠⠠	(46,6,23456)
extended left bracket		⠠⠠⠠	(4,6,12356)
extended right bracket		⠠⠠⠠	(4,6,23456)
extended left parenthesis		⠠⠠	(6,12356)
extended right parenthesis		⠠⠠	(6,23456)
extended left transcriber's enclosure		⠠⠠⠠	(46,6,235)
extended right transcriber's enclosure		⠠⠠⠠	(46,6,256)
extended vertical bar left and right		⠠⠠⠠	(4,6,1256)
lower left half bracket		⠠⠠⠠	(4,56,12356)
lower right half bracket		⠠⠠⠠	(4,56,23456)
upper left half bracket		⠠⠠⠠	(4,45,12356)
upper right half bracket		⠠⠠⠠	(4,45,23456)

**Examples for extended enclosures:**

$$\begin{cases} x + 2y = 6 \\ 2x - 2y = 7 \end{cases}$$

$$\begin{pmatrix} x + 2y = 8 \\ 2x - 3y = 2 \end{pmatrix}$$



**Determinants and Matrices**

Determinants and matrices are spatial arrangements, both in print and in braille. They contain arrays of numbers, variables, trigonometric functions, etc. They are readily recognized by their format in print. Here are some print examples of such arrays.

$$\begin{vmatrix} ab & cd \\ ac & ce \end{vmatrix}$$

$$\begin{vmatrix} \sin x & \cos y \\ -\cos x & \sin y \end{vmatrix}$$

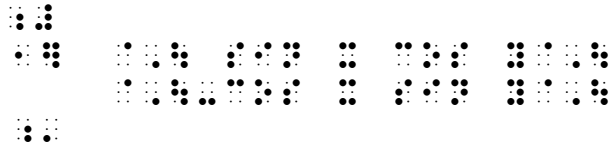
$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

**28.1.1 Rules for Determinants and Matrices**

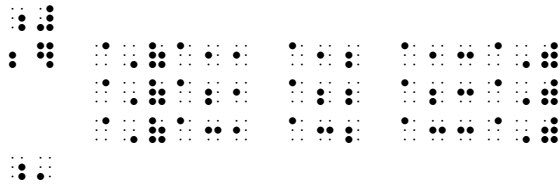
1. If the array has an identifier label, that label should be aligned with the top line of the array.
2. Ellipses in arrays, dots (6,6,3), represent missing elements in a single row. An entire missing row is represented by a row of dots 5. See problems number 3 and number 4 below for how they should look.
3. Make every effort to braille a matrix or determinant across a single block of lines. In problems involving more than one matrix or determinant, it may be necessary to place the matrices one above the other. This may mean that
  - a. An identifier must stand alone on a line by itself,
  - b. A preceding part of the math expression must be separated from the rest of the expression as shown in the third example below, or
  - c. Two determinants that are multiplied together must be separated.
4. Matrices and determinants are always brailled in a spatial format so the notational indicators are needed on the lines above and below them.

**Examples:**

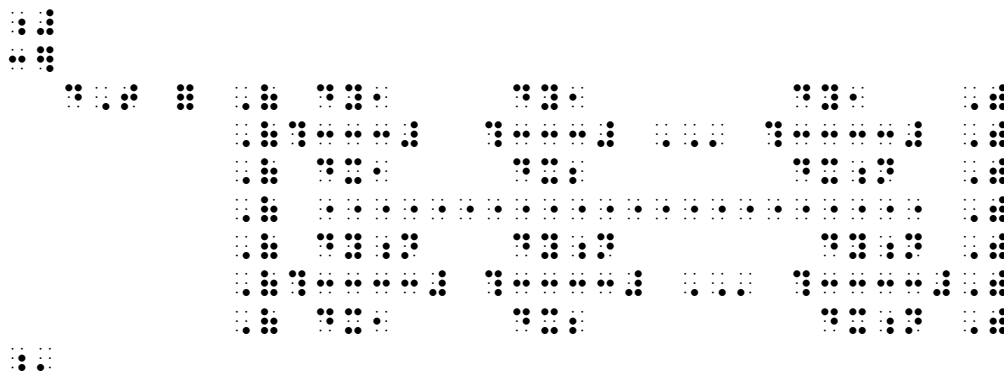
$$1. \begin{vmatrix} \sin x \cos y \\ -\cos x \sin y \end{vmatrix}$$



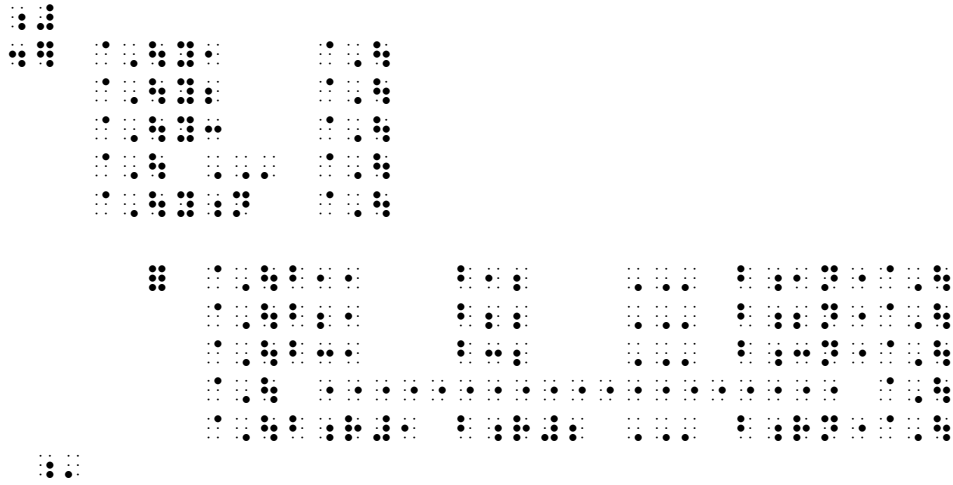
$$2. \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$



$$3. dT = \begin{pmatrix} \frac{dy_1}{dx_1} & \frac{dy_1}{dx_2} & \dots & \frac{dy_1}{dx_n} \\ \dots & \dots & \dots & \dots \\ \frac{dy_n}{dx_1} & \frac{dy_n}{dx_2} & \dots & \frac{dy_n}{dx_n} \end{pmatrix}$$



$$4. \quad \begin{vmatrix} y_1 \\ y_2 \\ y_3 \\ \dots \\ y_n \end{vmatrix} = \begin{vmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ b_{31} & b_{32} & \dots & b_{3n} \\ \dots & \dots & \dots & \dots \\ b_{r1} & b_{r2} & \dots & b_{rn} \end{vmatrix}$$



## Lesson 29

### Calculus and Logic and Set Theory

#### 29.1 In General

The field of calculus is one of the most important areas of mathematical study, especially in the field of engineering. Calculus, which at one time was introduced at the college level, is now being taught to high school juniors and seniors.

#### 29.2 Symbols Used in Calculus

Many special symbols are needed for expressing mathematical processes involving calculus. Here is a list of the most common calculus symbols. They are all notational.

Shape	Print	Braille	Dots
Absolute value		⠠	(4,1256)
Caret	^	⠠	(6,26)
Del (or nabla)	∇	⠠	(456,245)
Double vertical bar		⠠	(4,4,1256)
Factorial	!	⠠	(456,2346)
Infinity	∞	⠠	(4,236)
Integral	∫	⠠	(2346)
Large boldface pi	<b>π</b>	⠠	(456,6,1234)
Large sigma	<b>Σ</b>	⠠	(456,6,234)
Limit	lim	⠠	(123,24,134)
Logarithm	log	⠠	(123,135,1245)
Lower limit	<u>lim</u>	⠠	(146,123,24,134,156)
Natural logarithm	ln	⠠	(123,1345)
Norm		⠠	(4,4,1256)
Partial derivative	∂	⠠	(4,1246)
Upper limit	$\overline{\lim}$	⠠	(126,123,24,134,156)
Vertical bar		⠠	(4,1256)



### 29.3 Overscripts and Underscripts

In addition to subscripts and superscripts, mathematical notation permits stacked notation. In notation of this kind, the principal expression is modified by subsidiary expressions which are written below the principal expression, above the principal expression, or both. These subsidiary expressions are called underscripts and overscripts. The principal expression may contain more than one underscript, more than one overscript, or more than one of both. These are written at successively more remote levels relative to the principal expression.

When any expression is accompanied by some print notation above or below the principal expression, such notation must be properly represented in braille. This issue has been addressed to a limited degree by the use of subscripts, superscripts and the process called "modification". This does not cover all of the material that may appear "over" or "under" other expressions. The "modification" technique discussed in Lesson 26 was limited to a specific list of permissible modifiers.

The use of "overscripts and "underscripts" provides a technique for representing all of the "over-under" expressions that occur in scientific texts. This process should be used only when the "modification" process is not applicable. NUBS has a technique for treating such text, which occurs frequently in the use of the symbols for functions in this lesson. This process is especially useful in the proper representation of text that appears above and below the symbols listed in the list of Symbols Used in Calculus.

#### Indicators for Overscripts and Underscripts

Underscripts must be entered first.

First level underscript ⠨⠨ (56,12346)

First level overscript ⠨⠨ (45,12346)

- Intermediate indicator ⠨ (12346) to follow each script
- Final terminator ⠨ (246) to appear at the end of the final script.

Start the expression with a notational indicator, dots (56) and then the character that will have an underscript and or an overscript. Then the underscript indicator followed by the information in the underscript is entered. If there is an overscript to be entered, the intermediate terminator is placed after the underscript. The

overscript indicator followed by the overscript is entered and then the final terminator is entered. If there was not to be an overscript, the final terminator rather than the intermediate indicator would have been used after the underscript.

If there are more than one level of underscript or overscript, additional ⋮, dots (56) or ⋮, dots (45) indicators would be added for each subsequent level of underscript, or overscript. Whether there is one or there are several such prefixes, each sequence must be followed by the ⋮ (12346) character which is called the intermediate indicator. The intermediate indicator is used after each level of script that is not the last script on the principal expression. The final terminator follows the last script being used on any particular principal expression, to show that the end of the stack has been reached.

Keep in mind that when expressions are used as functions, a space is required before its argument so notational phrase indicators are required. Thus, some of the examples below will start with two notational indicators, dots (56,56) and end with dots (56,3).

When there are signs of comparison within a script, the usual spaces around that sign will be omitted.

### 29.3.1 Examples of underscripts and overscripts

Some of the symbols from the list of symbols used in calculus.

$$\lim_{x \rightarrow 0} f(x)$$

$$\lim_{x \rightarrow a^+} f(x)$$

Note in this example that the plus sign after the letter "a" is part of the underscript, as shown by the fact that the underscript has not yet been terminated.

$$\sum_{n=0}^{\infty} x_n$$



$$\int_0^2 x^2 dx = \frac{1}{3} x^3 \Big|_0^2 = \frac{1}{3} (8 - 0) = \frac{8}{3}$$



$$\int_1^2 \int_0^{x-1} y dy dx$$



$$\iint_D (x^2+y^2) dx dy$$



## 29.5 Logic and Set Theory

Logic and set theory are closely related branches of mathematics and thus share many symbols. Most of those symbols can be classified either as operation signs or as comparison signs, and have been included in their respective categories in the Mathematics Symbol Set, together with their tactile graphics. They are governed by the same rules that we have already encountered regarding other operation signs and comparison signs.

Note that the symbols for "meet" and "join" are top-to-bottom symmetric in print; they are likewise top-to-bottom symmetric in NUBS. The same observation holds for the signs for "union" and "intersection."

Note that the prefix  $\neg$  (46) denotes negation. It is placed either before the first character of the principal sign to achieve negation, or it replaces that first character.

The comparison signs in the Logic and Set Theory list have been duplicated in the Signs of Comparison list, and the operation signs in the Logic and Set Theory list have been duplicated in the Operation Signs list. Four entries in the Logic and Set Theory list are neither comparison signs nor operation signs.

### 29.5.1 Logic and Set Theory Characters

Shape	Print	Braille	Dots
Cartesian product	×	$\cdot\cdot$	(46,246)

contains the element	$\in$	$\begin{smallmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{smallmatrix}$	(4,4,24)
contains the subset	$\supset$	$\begin{smallmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{smallmatrix}$	(4,4,13456)
does not contain the element	$\notin$	$\begin{smallmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{smallmatrix}$	(46,4,24)
does not contain the subset	$\not\supset$	$\begin{smallmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{smallmatrix}$	(46,4,13456)
empty set	$\emptyset$	$\begin{smallmatrix} \cdot & \cdot \\ \cdot & \cdot \end{smallmatrix}$	(456,356)
for all	$\forall$	$\begin{smallmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{smallmatrix}$	(456,456,3)
global Cartesian product	$\times$	$\begin{smallmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{smallmatrix}$	(456,46,246)
global intersection	$\subset$	$\begin{smallmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{smallmatrix}$	(456,4,134)
global product		$\begin{smallmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{smallmatrix}$	(456,6,1234)
global sum		$\begin{smallmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{smallmatrix}$	(456,6,234)
global union	$\cup$	$\begin{smallmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{smallmatrix}$	(456,4,136)
if and only if	iff	$\begin{smallmatrix} \cdot & \cdot \\ \cdot & \cdot \end{smallmatrix}$	(4,123456)
implies	$\supset$	$\begin{smallmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{smallmatrix}$	(4,4,13456)
intersection	$\subset$	$\begin{smallmatrix} \cdot & \cdot \\ \cdot & \cdot \end{smallmatrix}$	(4,134)
is an element of	$\in$	$\begin{smallmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{smallmatrix}$	(4,4,15)
is implied by	$\cap$	$\begin{smallmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{smallmatrix}$	(4,4,12346)
is not an element of	$\notin$	$\begin{smallmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{smallmatrix}$	(46,4,15)
is not a subset of	$\not\subset$	$\begin{smallmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{smallmatrix}$	(46,4,12346)
is a proper subset of	$\subset$	$\begin{smallmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{smallmatrix}$	(456,4,12346)
is a subset of	$\subseteq$	$\begin{smallmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{smallmatrix}$	(4,4,12346)
join	$\wedge$	$\begin{smallmatrix} \cdot & \cdot \\ \cdot & \cdot \end{smallmatrix}$	(4,1236)
meet	$\vee$	$\begin{smallmatrix} \cdot & \cdot \\ \cdot & \cdot \end{smallmatrix}$	(4,1234)
there exists	$\exists$	$\begin{smallmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{smallmatrix}$	(456,456,26)
there exists uniquely	$\exists  $	$\begin{smallmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{smallmatrix}$	(456,46,26)
union	$\cup$	$\begin{smallmatrix} \cdot & \cdot \\ \cdot & \cdot \end{smallmatrix}$	(4,136)



## Appendix A Signs of Operation

minus - ∷ (36)

plus + ∷ (346)

multiplication asterisk \* ∷∷ (456,246)

multiplication times cross × ∷∷ (46,246)

multiplication times dot · ∷∷ (6,12456)

divided by ∷∷ (46,34)

slash ∷∷ (456,34)

begin displayed division ∷ (135)



## Appendix B Signs of Comparison

is equal to	⠠⠠⠠⠠	(123456)	=
is greater than	⠠⠠⠠⠠⠠	(4,345)	>
is less than	⠠⠠⠠⠠⠠	(4,126)	<
is not equal to	⠠⠠⠠⠠	(46,123456)	≠
approximately equal	⠠⠠⠠⠠	(456,35)	≈
contains the element	⠠⠠⠠⠠⠠	(4,4,24)	∑
contains the subset	⠠⠠⠠⠠	(4,4,13456)	⊃
does not contain the element	⠠⠠⠠⠠⠠	(46,4,24)	∉
does not include	⠠⠠⠠⠠	(46,4,13456)	⊄
greater than or equal	⠠⠠⠠⠠	(456,345)	≥
implies	⠠⠠⠠⠠	(4,4,13456)	⊃
includes	⠠⠠⠠⠠	(4,4,13456)	⊃
is an element	⠠⠠⠠⠠	(4,4,15)	∈
is a subset	⠠⠠⠠⠠	(4,4,12346)	⊆
is not an element	⠠⠠⠠⠠	(46,4,15)	∉
is not a subset	⠠⠠⠠⠠	(46,4,12346)	⊈
less than or equal	⠠⠠⠠⠠	(456,126)	≤
not equal to	⠠⠠⠠⠠	(46,123456)	≠
not greater than	⠠⠠⠠⠠	(46,345)	≯
not less than	⠠⠠⠠⠠	(46,126)	≧
is congruent to	⠠⠠⠠⠠⠠⠠	(126,123456,46,35)	≅



## Appendix C

### Fractions and Radicals

begin fraction ∴ (1456)

fraction bar ∴ (34) (Not a two-cell slash !)

Two-cell slash ∴ ∴ (456,34)

end fraction ∴ (3456)

#### Symbols used in Cancellation

Begin cancellation ∴ (12346)

End cancellation ∴ (246)

One-character cancellation ∴ (456)

#### Symbols used in complex fractions:

Open complex fraction ∴ ∴ (6,1456)

Complex fraction line ∴ ∴ (6,34)

Close complex fraction ∴ ∴ (6,3456)

Begin outermost fraction ∴ ∴ ∴ (6,6,1456)

Outermost fraction bar ∴ ∴ ∴ (6,6,34)

End outermost fraction ∴ ∴ ∴ (6,6,3456)

#### Symbols used in Radicals (Square Root)

Begin radical ∴ (345)

End radical ∴ (246)

#### Symbols used in Nested radicals:

Complexity indicator for

fractions and radicals ∴ (6)

	Begin	End
Innermost radical	∴	∴
Level 2 radical	∴ ∴	∴ ∴
Level 3 radical	∴ ∴ ∴	∴ ∴ ∴

Component of a radical ascender	∴	(456)
Component of spatial fraction bar	∵	(25)
Component of a spatial vinculum	∵	(25)
Last component of radical ascender	∴	(56)
Radical sign without vinculum	∴∴∴	(46,46,345)

## Appendix D



### Symbols used for Geometry

The three cells together stand for the shape.

Shape	Print	Braille	Dots
Angle	∠	⠠⠠⠠	(456,5,2346)
Arc - concave	⤿	⠠⠠⠠	(456,5,1)
- convex	⤾	⠠⠠⠠	(456,5,3)
Circle (ring)	◯	⠠⠠⠠	(456,5,14)
Diamond	◊	⠠⠠⠠	(456,5,145)
Ellipse	⬭	⠠⠠⠠	(456,5,15)
Equilateral triangle	△	⠠⠠⠠	(456,5,25)
Hexagon	⬡	⠠⠠⠠	(456,5,235)
Is parallel	∥	⠠⠠⠠	(456,5,123)
Is perpendicular	⊥	⠠⠠⠠	(456,5,1234)
Octagon	⬢	⠠⠠⠠	(456,5,236)
Oval	◌	⠠⠠⠠	(456,5,135)
Parallelogram	▭	⠠⠠⠠	(456,5,1245)
Pentagon	⬠	⠠⠠⠠	(456,5,26)
Quadrilateral	▭	⠠⠠⠠	(456,5,12345)
Rectangle	▭	⠠⠠⠠	(456,5,1235)
Rhombus	◊	⠠⠠⠠	(456,5,125)
Right angle	└	⠠⠠⠠	(456,5,1456)
Right triangle	▵	⠠⠠⠠	(456,5,1236)
Square	◻	⠠⠠⠠	(456,5,256)
Star	☆	⠠⠠⠠	(456,5,234)
Trapezoid	▭	⠠⠠⠠	(456,5,1356)
Triangle, generic	△	⠠⠠⠠	(456,5,2345)
Measure of angle	m∠	⠠⠠⠠⠠	(134,456,5,2346)

Adding dots (456) between the dot (5) and the shape designating letter makes the shape a solid filled in shape.

Adding dots (46) between the dot (5) and the shape designating letter makes the shape a shaded in shape.

**Examples:** solid rectangle ⠠⠠⠠⠠  (456,5,456,1235)  
shaded square ⠠⠠⠠⠠  (456,5,46,256)

An exception to the solid rule is the solid dot which is represented by the symbol for notational period. Solid dot • ⠠ (12456)

## General Modifiers

acute accent	´	⠠⠠	(4,35)
asterisk	*	⠠⠠	(456,246)
breve	˘	⠠⠠⠠	(456,5,1)
circumflex accent	ˆ	⠠⠠	(6,26)
diaeresis	¨	⠠⠠	(46,25)
dot	•	⠠	(12456)
grave accent	`	⠠⠠	(4,26)
hat	^	⠠⠠	(6,26)
left arrow	←	⠠⠠	(4,246)
macron (bar)	—	⠠	(156)
right arrow	→	⠠⠠	(4,135)
ring	◊	⠠⠠	(46,346)
tilde	~	⠠⠠	(46,35)
two-way arrow	↔	⠠⠠⠠	(4,123456)
umlaut	¨	⠠⠠	(46,25)

## Geometry Modifiers

full line	↔	⠠⠠	(4,123456)
half lines			
closed left	●→	⠠⠠⠠	(456,4,135)
closed right	←●	⠠⠠⠠	(456,4,246)
open left	○→	⠠⠠⠠	(46,4,135)
open right	←○	⠠⠠⠠	(46,4,246)
line segments			
left closed, right closed	●—●	⠠⠠⠠	(456,456,156)
left closed, right open	●—○	⠠⠠⠠	(456,46,156)
left open, right closed	○—●	⠠⠠⠠	(46,456,156)
left open, right open	○—○	⠠⠠⠠	(46,46,156)

## Appendix E

### Trigonometry Functions and Abbreviations

amplitude	amp
antilogarithm	antilog
arc	arc
argument	arg
cologarithm	colog
cosine + <i>isin</i>	cis
coversine	covers
curl	curl
determinant	det
dimension	dim
divergence	div
error function	erf
exponential	exp
exsecant	exsec
gradient	grad
haversine	hav
imaginary	im
infimum	inf
limit	lim
logarithm	log
lower limit	<u>lim</u> ⠠⠠⠠⠠⠠⠠
maximum	max
minimum	min
modulo	mod
modulus	mod
natural logarithm	ln
rank	rnk
real	re
supremum	sup
upper limit	<u>lim</u> ⠠⠠⠠⠠⠠⠠
versine	vers
arcsecant	arccsc
arccosine	arccos
arccotangent	arccot
arccotangent	arcctn

arcsecant	arcsec
arcsine	arcsin
arctangent	arctan
cosecant	csc
cosine	cos
cotangent	cot
cotangent	ctn
hyperbolic cosecant	csch
hyperbolic cosine	cosh
hyperbolic cotangent	coth
hyperbolic cotangent	ctnh
hyperbolic secant	sech
hyperbolic sine	sinh
hyperbolic tangent	tanh
inverse cosecant	$\text{csc}^{-1}$
inverse cosine	$\text{cos}^{-1}$
inverse cotangent	$\text{cot}^{-1}$
inverse cotangent	$\text{ctn}^{-1}$
inverse hyperbolic cosecant	$\text{csch}^{-1}$
inverse hyperbolic cosine	$\text{cosh}^{-1}$
inverse hyperbolic cotangent	$\text{coth}^{-1}$
inverse hyperbolic cotangent	$\text{ctnh}^{-1}$
inverse hyperbolic secant	$\text{sech}^{-1}$
inverse hyperbolic sine	$\text{sinh}^{-1}$
inverse hyperbolic tangent	$\text{tanh}^{-1}$
inverse secant	$\text{sec}^{-1}$
inverse sine	$\text{sin}^{-1}$
inverse tangent	$\text{tan}^{-1}$
radian	rad
secant	sec
sine	sin
tangent	tan

## Appendix F Signs of Grouping

### Angle brackets

left angle bracket    ⋮ ⋮ (4,126)    <

right angle bracket    ⋮ ⋮ (4,345)    >

### Barred signs

braces

left    ⋮ ⋮ (46,12356)    {

right    ⋮ ⋮ (46,23456)    }

brackets

left    ⋮ ⋮ (4,12356)    [

right    ⋮ ⋮ (4,23456)    ]

### Parentheses

left    ⋮ (12356)

right    ⋮ (23456)

### Transcriber's enclosures

left    ⋮ ⋮ (46,235)

right    ⋮ ⋮ (46,256)

### Vertical bar

left and right    ⋮ ⋮ (4,1256)

### Boldface signs

brackets

left    ⋮ ⋮ ⋮ (4,456,12356)    [

right    ⋮ ⋮ ⋮ (4,456,23456)    ]

vertical bar

left and right    ⋮ ⋮ ⋮ (4,456,1256)    |

### Double vertical bars

left and right    ⋮ ⋮ ⋮ (4,4,1256)    ||

## Extended signs

braces

left     ⠠⠠⠠⠠     (46,6,12356)

right    ⠠⠠⠠⠠     (46,6,23456)

brackets

left     ⠠⠠⠠⠠     (4,6,12356)

right    ⠠⠠⠠⠠     (4,6,23456)

parentheses

left   ⠠⠠⠠⠠   (6,12356)

right ⠠⠠⠠⠠   (6,23456)

## Half-brackets

lower left     ⠠⠠⠠⠠     (4,56,12356)

lower right    ⠠⠠⠠⠠     (4,56,23456)

upper left     ⠠⠠⠠⠠     (4,45,12356)

upper right    ⠠⠠⠠⠠     (4,45,23456)

# Appendix G

## Calculus Symbols

absolute value	⠠⠨⠠⠨	(4,1256)	
del	⠠⠨⠠⠨	(456,245)	□
double vertical bar	⠠⠨⠠⠨⠠⠨⠨	(4,4,1256)	
infinity	⠠⠨⠠⠨	(4,236)	∞
integral	⠠⠨	(2346)	∫
large boldface pi	⠠⠨⠠⠨⠠⠨⠨	(456,6,1234)	π
large boldface sigma	⠠⠨⠠⠨⠠⠨⠨	(456,6,234)	Σ
limit	⠠⠨⠠⠨⠠⠨⠨		lim
logarithm	⠠⠨⠠⠨⠠⠨⠨		log
lower limit	⠠⠨⠠⠨⠠⠨⠠⠨⠠⠨⠨		<u>lim</u>
nabla	⠠⠨⠠⠨	(456,245)	□
natural logarithm	⠠⠨⠠⠨		ln
norm	⠠⠨⠠⠨⠠⠨⠨	(4,4,1256)	
partial derivative	⠠⠨⠠⠨	(4,1246)	∂
prime	⠠⠨	(3)	′
upper limit	⠠⠨⠠⠨⠠⠨⠠⠨⠠⠨⠨		<u>lim</u>
vertical bar	⠠⠨⠠⠨	(4,1256)	
Weierstrass $p$	⠠⠨⠠⠨⠠⠨⠨	(46,5,1234)	



# Appendix H

## Greek Alphabet

<b>Lower Case</b>			<b>Upper Case</b>		
Symbol	Name	Braille	Symbol	Name	Braille
$\alpha$	alpha	⠠⠠	A	alpha	⠠⠠
$\beta$	beta	⠠⠨	B	beta	⠠⠨
$\gamma$	gamma	⠠⠢	Γ	gamma	⠠⠢
$\delta$	delta	⠠⠴	Δ	delta	⠠⠴
$\epsilon$	epsilon	⠠⠢⠠	E	epsilon	⠠⠢⠠
$\zeta$	zeta	⠠⠵	Z	zeta	⠠⠵
$\eta$	eta	⠠⠨⠠	H	eta	⠠⠨⠠
$\theta$	theta	⠠⠴⠠	Θ	theta	⠠⠴⠠
$\iota$	iota	⠠⠠⠠	I	iota	⠠⠠⠠
$\kappa$	kappa	⠠⠠⠨	K	kappa	⠠⠠⠨
$\lambda$	lambda	⠠⠠⠴	Λ	lambda	⠠⠠⠴
$\mu$	mu	⠠⠠⠢	M	mu	⠠⠠⠢
$\nu$	nu	⠠⠠⠵	N	nu	⠠⠠⠵
$\xi$	xi	⠠⠠⠵⠠	Ξ	xi	⠠⠠⠵⠠
$\omicron$	omicron	⠠⠠⠵⠠	O	omicron	⠠⠠⠵⠠
$\pi$	pi	⠠⠠⠵⠠	Π	pi	⠠⠠⠵⠠
$\rho$	rho	⠠⠠⠵⠠	P	rho	⠠⠠⠵⠠
$\sigma$	sigma	⠠⠠⠵⠠	Σ	sigma	⠠⠠⠵⠠
$\tau$	tau	⠠⠠⠵⠠	T	tau	⠠⠠⠵⠠
$\upsilon$	upsilon	⠠⠠⠵⠠	Y	upsilon	⠠⠠⠵⠠
$\phi$	phi	⠠⠠⠵⠠	Φ	phi	⠠⠠⠵⠠
$\chi$	chi	⠠⠠⠵⠠	X	chi	⠠⠠⠵⠠
$\psi$	psi	⠠⠠⠵⠠	Ψ	psi	⠠⠠⠵⠠
$\omega$	omega	⠠⠠⠵⠠	Ω	omega	⠠⠠⠵⠠



# Appendix I

## Logic and Set Theory

Cartesian product	⋮⋮	(46,246)	×
contains the element	⋮⋮⋮	(4,4,24)	∈
contains the subset	⋮⋮⋮	(4,4,13456)	⊃
does not contain the element	<u>⋮⋮⋮</u>	(46,4,24)	∉
does not contain the subset	⋮⋮⋮	(46,4,13456)	⊄
empty set	⋮⋮	(456,356)	∅
for all	⋮⋮⋮	(456,456,3)	∀
global Cartesian product	⋮⋮⋮	(456,46,246)	×
global intersection	⋮⋮⋮	(456,4,134)	⊂
global product	⋮⋮⋮	(456,6,1234)	
global sum	⋮⋮⋮	(456,6,234)	
global union	⋮⋮⋮	(456,4,136)	∪
if and only if	⋮⋮	(4,123456)	iff
implies	⋮⋮⋮	(4,4,13456)	⊃
intersection	⋮⋮	(4,134)	⊂
is an element of	⋮⋮⋮	(4,4,15)	∈
is implied by	⋮⋮⋮	(4,4,12346)	⊂
is not an element of	⋮⋮⋮	(46,4,15)	∉
is not a subset of	⋮⋮⋮	(46,4,12346)	⊄
is a proper subset of	⋮⋮⋮	(456,4,12346)	⊂
is a subset of	⋮⋮⋮	(4,4,12346)	⊆
join	⋮⋮	(4,1236)	∧
meet	⋮⋮	(4,1234)	∨

there exists	⋮⋮⋮	(456,456,26)	∃
there exists uniquely	⋮⋮⋮	(456,46,26)	∃
union	⋮⋮	(4,136)	∪

## Appendix J Miscellaneous Symbols

### Arrows

northeast		(4,45,135)	↗
northwest		(4,45,246)	↖
northwest-southeast		(4,56,123456)	↔
southeast		(4,56,135)	↘
southwest		(4,56,246)	↙
southwest-northeast		(4,45,123456)	↕
ampersand		(4,12346)	&
apostrophe		(3)	'
asterisk		(456,246)	*
at sign		(4,1)	@
backslash		(456,16)	\
caret		(6,26)	^
colon		(25)	:
comma		(16)	,
crosshatch		(46,3456)	#
dollar sign		(4,234)	\$
equals		(123456)	=
exclamation		(235)	!
grave accent		(4,26)	`
hyphen		(36)	-
percent sign		(46,356)	%
period		(12456)	.
plus		(346)	+
question mark		(236)	?
quote		(6,2356)	“
semicolon		(23)	;
slash		(456,34)	/
space		(no dots)	
tilde		(46,35)	~
underscore		(6,36)	_
vertical bar		(4,1256)	

begin upper-case passage	⠠⠠⠠⠠	(6,6,56)
end upper-case passage	⠠⠠	(6,3)
upper case, one letter	⠠	(6)
upper case, one word	⠠⠠	(6,6)
no boundary	⠠	(4)
overscript	⠠⠠	(45,12346)
overstrike	⠠⠠	(5,12346)
runover	⠠	(456)
shaded shape	⠠	(46)
solid shape	⠠	(456)
underscript	⠠⠠	(56,12346)

### Computer and Calculator

begin keytop legend	⠠	(246)
begin screen display	⠠	(12346)
runover indicator	⠠	(456)

### Punctuation

ellipsis, vertical	⠠⠠	(456,2)
--------------------	----	---------

### Special Letters

#### Crossed

b	⠠⠠	(4,12)
d	⠠⠠	(4,145)
h	⠠⠠	(4,125)
λ (lambda)	⠠⠠⠠	(46,5,123)

### Global Operation Signs

global product	⠠⠠⠠	(456,6,1234)	Π
global sum	⠠⠠⠠	(456,6,234)	Σ

### Hebrew

aleph	⠠⠠	(46,2)	א
-------	----	--------	---

### Number Sets

complex	⠠⠠⠠	(456,456,14)
integers	⠠⠠⠠	(456,456,24)
natural	⠠⠠⠠	(456,456,1345)
rational	⠠⠠⠠	(456,456,1356)
real	⠠⠠⠠	(456,456,1235)

### Obsolete Greek

koph or koppa	⠠⠠⠠	(46,5,12345)
sampi	⠠⠠⠠	(46,5,156)

## Variant Greek

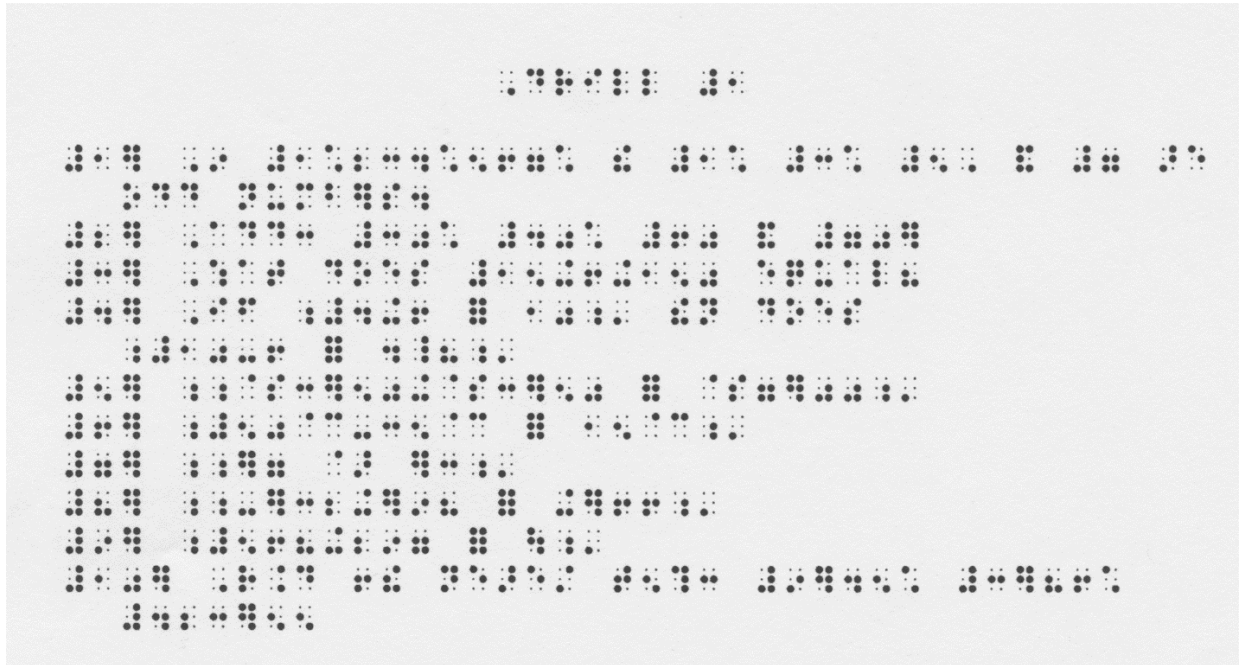
alpha	⠠⠠⠠⠠	(46,5,1)
beta	⠠⠠⠠⠠⠠	(46,5,12)
theta	⠠⠠⠠⠠⠠⠠	(46,5,1456)
sigma	⠠⠠⠠⠠⠠	(46,5,234)
phi	⠠⠠⠠⠠⠠	(46,5,124)



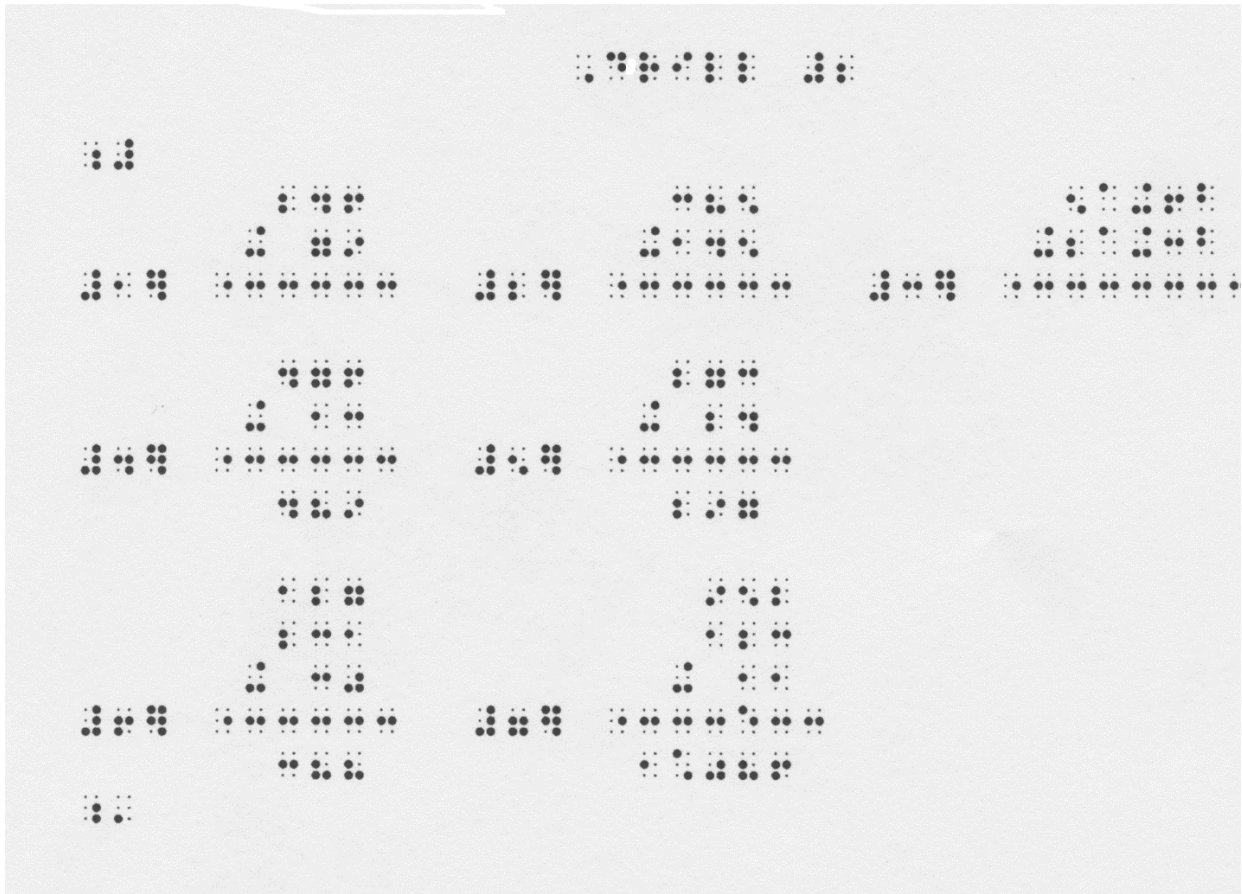
# Appendix K

## Braille Answers for Drills

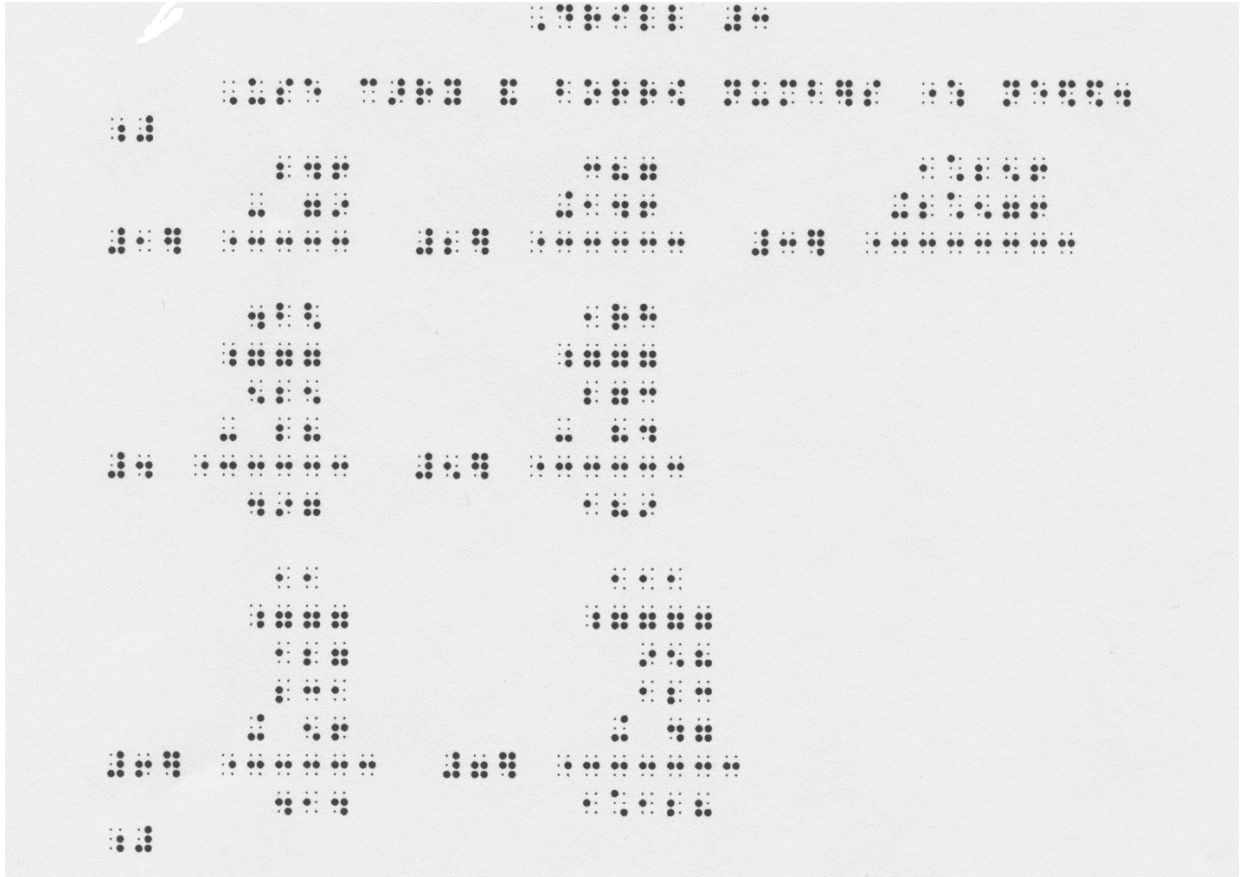
### Drill #1



# Drill #2



# Drill #3



Braille characters arranged in two lines, likely representing a specific word or phrase.





⠠⠠⠠

⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

⠠⠠

⠠⠠⠠

⠠⠠⠠

⠠⠠⠠

⠠⠠⠠

⠠⠠

⠠⠠⠠











# Drill #10

1. The first row contains a series of 15 groups of dots, each group consisting of a 2x2 grid of dots. The groups are arranged in a sequence that forms a pattern of dots.

2. The second row contains a series of 15 groups of dots, each group consisting of a 2x2 grid of dots. The groups are arranged in a sequence that forms a pattern of dots.

3. The third row contains a series of 15 groups of dots, each group consisting of a 2x2 grid of dots. The groups are arranged in a sequence that forms a pattern of dots.

4. The fourth row contains a series of 15 groups of dots, each group consisting of a 2x2 grid of dots. The groups are arranged in a sequence that forms a pattern of dots.

5. The fifth row contains a series of 15 groups of dots, each group consisting of a 2x2 grid of dots. The groups are arranged in a sequence that forms a pattern of dots.

6. The sixth row contains a series of 15 groups of dots, each group consisting of a 2x2 grid of dots. The groups are arranged in a sequence that forms a pattern of dots.

7. The seventh row contains a series of 15 groups of dots, each group consisting of a 2x2 grid of dots. The groups are arranged in a sequence that forms a pattern of dots.

8. The eighth row contains a series of 15 groups of dots, each group consisting of a 2x2 grid of dots. The groups are arranged in a sequence that forms a pattern of dots.

9. The ninth row contains a series of 15 groups of dots, each group consisting of a 2x2 grid of dots. The groups are arranged in a sequence that forms a pattern of dots.

10. The tenth row contains a series of 15 groups of dots, each group consisting of a 2x2 grid of dots. The groups are arranged in a sequence that forms a pattern of dots.





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